CISC-102 FALL 2017

HOMEWORK 7 SOLUTIONS

Problems

(1) Find all Natural numbers between 1 and 50 that are congruent to 4 (mod 11).
4, 15, 26, 37, 48. You can verify that 11 | (4 − 4), 11 | (15 − 4), 11 | (26 − 4), 11 | (37 − 4),
and 11 | (48 − 4).

(2) Find two Natural numbers a and b such that 2a ≡ 2b (mod 6),
but a ≢ b (mod 6).
You can solve this problem using trial and error. A good place to start is a = 1 so we have 2(1) ≡ 2 (mod 6),
The next Natural number that is congruent to 2 (mod 6) is 8. So setting b = 4 gives us 2(1) ≡ 2(4) (mod 6), and
1 ≢ 4 (mod 6).

(3) Prove that if a ≡ b (mod m) and c ≡ d (mod m) then a − c ≡ b − d (mod m).

Proof. a ≡ b (mod m) and c ≡ d (mod m) respectively imply:
b − a = pm and d − c = qm where p and q are integers.
Therefore we have:

\[(b - a) - (d - c) = pm - qm = m(p - q)\]

Therefore we can conclude that m | (b − a) − (d − c) so a − c ≡ b − d (mod m).

(4) Write out each of the 5 residue classes (mod 5) for integers in the range -10 to 10.

\[0]_5 = \{-10, -5, 0, 5, 10\}\n\[1]_5 = \{-9, -4, 1, 6\}\n\[2]_5 = \{-8, -3, 2, 7\}\n\[3]_5 = \{-7, -2, 3, 8\}\n\[4]_5 = \{-6, -1, 4, 9\}\n
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