CISC-102 WINTER 2020

HOMEWORK 9
SOLUTIONS

Problems

(1) Use a truth table to verify that the proposition $p \vee \neg(p \wedge q)$ is a tautology, that is, the expression is true for all values of p and q.

p	q	$p \wedge q$	$\neg(p \wedge q)$	$p \vee \neg(p \wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

(2) Use a truth table to verify that the proposition $(p \wedge q) \wedge \neg(p \vee q)$ is a contradiction, that is, the expression is false for all values of p and q.

p	q	$p \wedge q$	$p \vee q$	$\neg(p \vee q)$	$(p \wedge q) \wedge \neg(p \vee q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

(3) Use a truth table to show that $p \vee q \equiv \neg(\neg p \wedge \neg q)$.

p	q	$\neg p$	$\neg q$	$p \vee q$	$\neg p \wedge \neg q$	$\neg(\neg p \wedge \neg q)$
T	T	F	F	T	F	T
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	F	T	F

(4) Show that the following argument is valid.

$$p \to q, \neg q \vdash \neg p$$

We need to show that $[(p \to q) \wedge \neg q] \to \neg p$ is a tautology, and we do so using a truth table as follows:

$\neg p$	p	q	$\neg q$	$p \to q$	$(p \to q) \wedge \neg q$	$[(p \to q) \wedge \neg q] \to \neg p$
F	T	F	T	F	F	T
F	T	T	F	T	F	T
T	F	T	F	T	F	T
T	F	F	T	T	T	T

(5) Let A = {1,2,3,4,5}. Determine the truth value of each of the following statements.

(a) $(\exists x \in A)(x + 2 = 7)$
This is true with $x = 5$.

(b) $(\forall x \in A)(x + 2 < 8)$
This is true, because

$$(1 + 2 < 8) \wedge (2 + 2 < 8) \wedge (3 + 2 < 8) \wedge (4 + 2 < 8) \wedge (5 + 2 < 8).$$

(c) $(\exists x \in A)(x + 3 < 2)$
This is false because:

$$(1 + 3 \not< 2) \wedge (2 + 3 \not< 2) \wedge (3 + 3 \not< 2) \wedge (4 + 3 \not< 2) \wedge (5 + 3 \not< 2).$$

(d) $(\forall x \in A)(x + 3 \leq 9)$
This is true, because

$$(1 + 3 \leq 9) \wedge (2 + 3 \leq 9) \wedge (3 + 3 \leq 9) \wedge (4 + 3 \leq 9) \wedge (5 + 3 \leq 9).$$