CISC-102 FALL 2016

HOMEWORK 10

Please work on these problems and be prepared to share your solutions with classmates in class next Friday. Assignments will **not** be collected for grading.

Readings

Read chapter 4 of *Schaum's Outline of Discrete Mathematics.*
Read section 3.1, 3.5 and 3.6 of *Discrete Mathematics Elementary and Beyond.*

Problems

(1) Pascal's triangle is symmetric about its central column. That is for an odd number of entries in a row (as in row 8) the same numbers are found when moving backward and forward from the central value 70. A row with an even number of entries such as row 5: 1 5 10 10 5 1, exhibits a similar pattern without a unique central value. Explain why Pascal's triangle exhibits this symmetry, using one of the binomial coefficient identities that we saw this week in class.
(2) Use a truth table to verify that the proposition $p \vee \neg(p \wedge q)$ is a tautology, that is, the expression is true for all values of p and q.
(3) Use a truth table to verify that the proposition $(p \wedge q) \wedge \neg(p \vee q)$ is a contradiction, that is, the expression is false for all values of p and q.
(4) Use a truth table to show that $p \vee q \equiv \neg(\neg p \wedge \neg q)$
(5) Show that the following argument is valid.

$$p \rightarrow q, \neg q \vdash \neg p$$

(6) Let A = {1,2,3,4,5}. Determine the truth value of each of the following statements.
   (a) $(\exists x \in A)(x + 2 = 7)$
   (b) $(\forall x \in A)(x + 2 < 8)$
   (c) $(\exists x \in A)(x + 3 < 2)$
   (d) $(\forall x \in A)(x + 3 \leq 9)$
(7) Let $A = \{1, 2, 3, 4, 5\}$. And let $(x, y) \in A^2$, be the domain of the propositions given below. Determine the truth value of the following statements.
   (a) $\exists x \forall y, x^2 < y + 1$
   (b) $\forall x \exists y, x^2 < y + 1$