Please work on these problems and be prepared to share your solutions with classmates in class next Monday. Assignments will not be collected for grading.

**Readings**

Read sections 1.8 of *Schaum’s Outline of Discrete Mathematics*.
Read section 2.1 of *Discrete Mathematics Elementary and Beyond*.

**Problems**

1. Prove using mathematical induction that the sum of the first \( n \) natural numbers is equal to \( \frac{n(n+1)}{2} \). This can also be stated as:
   
   Prove that the proposition \( P(n) \),
   
   \[ \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \]
   
   is true for all \( n \in \mathbb{N} \)

2. Prove using mathematical induction that the proposition \( P(n) \),
   
   \[ \sum_{i=1}^{n} \frac{1}{2^i} = 1 - \frac{1}{2^n} \]
   
   is true for all \( n \in \mathbb{N} \)

3. Prove using mathematical induction that the proposition \( P(n) \)
   
   \[ n! \leq n^n \]
   
   is true for all \( n \in \mathbb{N} \).

4. Let \( S \) be a set of \( n \) elements, such that \( a \in S \). Show that there are the same number of subsets of \( S \) that do contain \( a \) as there are subsets of \( S \) that do not contain \( a \).