CISC-102 FALL 2015

HOMEWORK 3

Please work on these problems and be prepared to share your solutions with classmates in class next Monday. Assignments will **not** be collected for grading.

Readings

Read sections 1.8 of *Schaum's Outline of Discrete Mathematics.*
Read section 2.1 of *Discrete Mathematics Elementary and Beyond.*

Problems

(1) Prove using mathematical induction that the sum of the first n natural numbers is equal to $\frac{n(n+1)}{2}$. This can also be stated as:
Prove that the proposition P(n),

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

is true for all $n \in \mathbb{N}$

(2) Prove using mathematical induction that the proposition P(n),

$$\sum_{i=1}^{n} \frac{1}{2^i} = 1 - \frac{1}{2^n}$$

is true for all $n \in \mathbb{N}$

(3) Prove using mathematical induction that the proposition P(n)

$$n! \leq n^n$$

is true for all $n \in \mathbb{N}$.

(4) Let S be a set of n elements, such that $a \in S$. Show that there are the same number of subsets of S that do contain a as there are subsets of S that do not contain a.