CISC-102 FALL 2016

HOMEWORK 8

Please work on these problems and be prepared to share your solutions with classmates in class next week. Assignments will **not** be collected for grading.

Readings

Read sections 5.3, 5.4, 5.5 5.6 and 5.7 of *Schaum's Outline of Discrete Mathematics.*
Read section 3.1 and 3.2 of *Discrete Mathematics Elementary and Beyond.*

Problems

(1) Let S be a finite subset of the positive integers. What is the smallest value for $|S|$ that guarantees that at least two elements of $x, y \in S$ that have the same remainder when divided by 100. HINT: Use the pigeon hole principle.
(2) What is the number of ways to colour n identical objects with 2 colours, so that each colour is used at least once?
(3) What is the number of ways to colour n different objects with 2 colours, so that each colour is used at least once?
(4) How many 5 card hands are there (unordered selection from a standard 52 card deck) that consist of a single pair of the same value, and three other cards of different values? Two possible examples are:

$$2\heartsuit, 2\diamondsuit, 7\clubsuit, 9\diamondsuit 3\heartsuit \text{ and } A\heartsuit, A\clubsuit, 4\diamondsuit, 6\diamondsuit 3\heartsuit$$

(5) A skip straight is 5 cards that are in consecutive order, skipping every second rank (for example 3-5-7-9-J). How many 5 card hands are there (unordered selection from a standard 52 card deck) that form a skip straight?
(6) Use the binomial theorem to expand the product $(x+y)^6$.
(7) Show that

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \cdots + \binom{n}{n} = 0$$

HINT: Use the Binomial theorem.