Please work on these problems and be prepared to share your solutions with classmates in class next week. Assignments will not be collected for grading.

Read sections 5.3, 5.4, 5.5, 5.6 and 5.7 of Schaum’s Outline of Discrete Mathematics. Read section 3.1 and 3.2 of Discrete Mathematics Elementary and Beyond.

Problems

(1) Let \( S \) be a finite subset of the positive integers. What is the smallest value for \( |S| \) that guarantees that at least two elements of \( x, y \in S \) that have the same remainder when divided by 100. HINT: Use the pigeon hole principle.

(2) What is the number of ways to colour \( n \) identical objects with 2 colours, so that each colour is used at least once?

(3) What is the number of ways to colour \( n \) different objects with 2 colours, so that each colour is used at least once?

(4) How many 5 card hands are there (unordered selection from a standard 52 card deck) that consist of a single pair of the same value, and three other cards of different values? Two possible examples are:

\[ 2 \heartsuit, 2 \diamondsuit, 7 \clubsuit, 9 \heartsuit \text{ and } A \heartsuit, A \clubsuit, 4 \diamondsuit, 6 \diamondsuit, 3 \heartsuit \]

(5) A skip straight is 5 cards that are in consecutive order, skipping every second rank (for example 3-5-7-9-J). How many 5 card hands are there (unordered selection from a standard 52 card deck) that form a skip straight?

(6) Use the binomial theorem to expand the product \((x + y)^6\).

(7) Show that

\[
\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \cdots + \binom{n}{n} = 0
\]

HINT: Use the Binomial theorem.