HOMEWORK 9

Readings

Read sections 5.3 of *Schaum's Outline of Discrete Mathematics*.
Read sections 3.5, 3.6 of *Discrete Mathematics Elementary and Beyond*.

Problems

(1) Consider the equation

(1)
$$\sum_{i=0}^{2} \binom{3}{i}\binom{2}{2-i} = \binom{5}{2}.$$

    (a) Use algebraic manipulation to prove that the left hand and right hand sides of equation (1) are in fact equal.

    (b) Use a counting argument to prove that the left hand and right hand sides of equation (1) are in fact equal.

(2) Now consider a generalization of the previous equation.

(2)
$$\sum_{i=0}^{k} \binom{m}{i}\binom{n}{k-i} = \binom{m+n}{k}.$$

Use a counting argument to prove that the left hand and right hand sides of equation (2) are in fact equal.

(3) In the notes for Week 9 you will find Pascal's triangle worked out for rows 0 to 8. The numbers in row 8 are 1 8 28 56 70 56 28 8 1. Work out the values of rows 9 and 10 of Pascal's triangle with the help of the equation:

(3)
$$\binom{n-1}{k} + \binom{n-1}{k-1} = \binom{n}{k}.$$

(4) Show that $\binom{n}{0} = \binom{n-1}{0}$, and that $\binom{n-1}{n-1} = \binom{n}{n}$ by an algebraic argument as well as a counting argument.