CISC-102 Fall 2016

Quiz 1

September 26, 2016

Solutions

Student ID: _____

Read the questions carefully. Please clearly state any assumptions that you make that are not explicitly stated in the question.

Please answer all questions in the space provided. Use the back of pages for scratch work. There are 4 pages to this quiz. Note that (x) denotes the question is worth x points.

CALCULATORS ARE NOT PERMITTED.

1. Rewrite the following statements using set builder notation, and then describe the set by listing its members.

   (a) (2) A is the set of natural numbers greater than 5 and less than 6.

   $$A = \{x : x \in \mathbb{N}, 5 < x < 6\} \qquad A = \emptyset$$

   (b) (2) B is the set of integers x such that $x^2 - 9 = 0$.

   $$B = \{x : x \in \mathbb{Z}, x^2 - 9 = 0\} \qquad B = \{3, -3\}$$

   (c) (2) C is the set of natural numbers such that $x + 7 \leq 10$

   $$C = \{x : x \in \mathbb{N}, x + 7 \leq 10\}$$
   $$C = \{1, 2, 3\}$$

2. Consider the following sets X = {1,2,3,4}, Y = {3,4,5,6}, Z = {1,2,3,4,5,6}.

(a) (4) Is {X,Y} a partition of Z? Explain your answer.

No, because $X \cap Y \neq \emptyset$

(b) (4) What are the values $|X|, |Y|, |X| + |Y|, |X \cup Y|$?

$|X| = 4$ $|Y| = 4$ $|X| + |Y| = 8$ $|X \cup Y| = 6$

(c) (4) Let E be a set that is a subset of $X \cap Y \cap Z$ with the most elements. What are the elements of E?

$E = \{3, 4\}$

(d) (4) Let F be a set that is a subset of $X \cup Y \cup Z$ with the fewest elements. What are the elements of F?

$F = \emptyset$

(e) (4) List the elements of the power set, P(W), where W = X ∩ Y.

$\{ \emptyset, \{3\}, \{4\}, \{3,4\} \}$

3. (8) A survey received 100 responses from people on what types of pets they owned. Sixty of the respondents said they had dogs, forty said they had cats, and twenty five said that they had neither cats nor dogs.

Use the principle of inclusion and exclusion to determine the number of respondents who have both cats and dogs. Explain how you arrived at your answer. You may use a Venn diagram to support your reasoning.

Let D = dog owner
 C = cat owners

$$|D| = 60 \quad |C| \ 40$$

$$|D \cup C| = 75$$

by the P.I.E.

$$75 = 60 + 40 - |D \cap C|$$

so $\quad |D \cap C| = 25$

4. (4) Determine whether these statements are true or false, and if the statement is false provide a short sentence explaining why.

(a) (2) $0 \in \emptyset$

F $\emptyset$ has no elements

(b) (2) $\emptyset \in \{0\}$

F $\emptyset \subseteq \{0\}$ but its not an element of $\{0\}$.

(c) (2) $\{0\} \subseteq \emptyset$

F $\emptyset$ has no elements and only one subset, $\emptyset$.

(d) (2) $\{0\} \subset \emptyset$

F see part c

(e) (2) $\emptyset \subset \emptyset$

F $\emptyset \subseteq \emptyset$ but $\emptyset$ is not a proper subset of $\emptyset$

(f) (2) $\emptyset \subseteq \emptyset$

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(g) (2) $\emptyset = 0$

F $\emptyset$ is a set not a number

(h) (2) $\emptyset \subset \{0\}$

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