CISC-102
Fall 2016

Quiz 4

November 29, 2016

Student ID: Solutions

Read the questions carefully. Please clearly state any assumptions that you make that are not explicitly stated in the question.

Please answer all questions in the space provided. Use the back of pages for scratch work. NO CALCULATORS. There are 5 pages to this quiz. Note that (x) denotes the question is worth x points.

1. Here are the first 6 rows of Pascal's triangle, that is row 0, to row 5.

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}
$$

(a) (2) Write out the next row, that is row 6, of Pascal's triangle.

1 6 15 20 15 6 1

(b) (4) Re-write the first six rows of Pascal's triangle using binomial coefficients.

$$\binom{0}{0}$$

$$\binom{1}{0} \quad \binom{1}{1}$$

$$\binom{2}{6} \quad \binom{2}{1} \quad \binom{2}{2}$$

$$\binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$$

$$\binom{4}{0} \quad \binom{4}{1} \quad \binom{4}{2} \quad \binom{4}{3} \quad \binom{4}{4}$$

$$\binom{5}{0} \quad \binom{5}{1} \quad \binom{5}{2} \quad \binom{5}{3} \quad \binom{5}{4} \quad \binom{5}{5}$$

2. Consider the elements of Pascal's triangle that are in boxes.

$$\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & \boxed{1} & & 1 & & & & \\
 & & & 1 & & \boxed{2} & & 1 & & & \\
 & & 1 & & 3 & & \boxed{3} & & 1 & & \\
 & 1 & & 4 & & 6 & & \boxed{4} & & 1 & \\
1 & & 5 & & 10 & & \boxed{10} & & 5 & & 1
\end{array}$$

(a) (6) It appears that the boxed numbers in row 1 to row 4 have the value of the row number. What would be the binomial coefficient for the entry along this diagonal in row k, and explain why this binomial coefficient is equal to k. You should provide two explanations, one that is algebraic , and the other using a counting argument.

$\binom{1}{0}$, $\binom{2}{1}$, $\binom{3}{2}$, $\binom{4}{3}$

in general $\binom{n}{n-1} = \frac{n!}{(n-1)!\,1!} = n$

There are n ways to choose $n-1$ objects from a set of n objects.

(b) (6) Observe that the boxed number in row 5 (this is the 4th entry in the row) is the sum of the boxed numbers in rows 1 to 4. A generalization of this observation leads to the formula:

$$\sum_{m=1}^{n} \binom{m}{m-1} = \binom{n+1}{n-1}$$

Use a proof by induction (the first form of induction is fine) to prove that the equations holds for all natural numbers n.

There are two ways to prove this.

(1) Observe that

$$\sum_{m=1}^{n} \binom{m}{m-1} = \binom{n+1}{n-1}$$

is equivalent to

$$\sum_{m=1}^{n} m = \frac{(n+1)(n)}{2}$$

- Now apply induction

(2) Prove directly using induction

Base: $n=1$ $\quad \binom{1}{0} = \binom{2}{0} = 1$

Ind Hyp.: Assume $\sum_{m=1}^{k} \binom{k}{k-1} = \binom{k+1}{k-1}$ for a fixed $k \geq 1$.

Ind Step:

$$\sum_{m=1}^{k+1} \binom{m}{m-1} = \binom{k+1}{k} + \sum_{m=1}^{k} \binom{m}{m-1}$$

$$= \binom{k+1}{k} + \binom{k+1}{k-1}$$

$$= \binom{k+2}{k}$$

Pf

3. (6) Fill in the following truth table

p	q	$p \wedge q$	$q \vee p$	$(p \wedge q) \rightarrow (q \vee p)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

This is a taughtology.

4. (6) Consider the logical argument

$$(p \vee q), ((\neg p) \vee r) \vdash (q \vee r).$$

Complete the truth table below, adding columns as needed to determine whether the argument is valid or not. After you have completed the table explain your conclusion in a sentence or two.

p	q	r	1: $p \vee q$	2: $\neg p \vee r$	3: $1 \wedge 2$	4: $q \vee r$	$3 \to 4$
F	F	F	F	T	F	F	T
F	F	T	F	T	F	T	T
F	T	F	T	T	T	T	T
F	T	T	T	T	T	T	T
T	F	F	T	F	F	F	T
T	F	T	T	T	T	T	T
T	T	F	T	F	F	T	T
T	T	T	T	T	T	T	T

$(p \vee q) \wedge (\neg p \vee r) \to (q \vee r)$ is a tautology, so the argument is valid.