CISC-102 Fall 2016

Quiz 2

October 18, 2016

Student ID: ____________________

Read the questions carefully. Please clearly state any assumptions that you make that are not explicitly stated in the question. Please answer all questions in the space provided. Use the back of pages for scratch work. There are 4 pages to this quiz. Note that (x) denotes the question is worth x points.

CALCULATORS ARE NOT PERMITTED.

1. Consider a function f from the set A to B where A $= \{x : x \in \mathbb{R}, 1 \leq x \leq 10\}$, and B $= \{x : x \in \mathbb{N}, 1 \leq x \leq 10\}$, and the mapping is defined as

   $f(x) =$ greatest element of B that is less than or equal to x. This is a special case of the *floor* function defined over a reduced domain and range. The function simply takes a real number in the closed interval $[1 \ldots 10]$ and rounds down to the closest integer. For example $f(2.3456) = 2$, $f(1.99999) = 1$, $f(7) = 7$.

   (a) (2) Is this function one-to one? Explain your answer.

   (b) (2) Is this function onto? Explain your answer.

   (c) (2) Is this function a bijection? Explain your answer.

2. Consider a relation Abs that is a subset of $\mathbb{Z} \times \mathbb{Z}$, such $\{(a, b) : a, b \in \mathbb{Z}, b = |a|\}$. This is the absolute value function, but since functions are a special case of relations we can consider Abs as a relation.

   (a) (2) Is this relation reflexive? Explain your answer?

   (b) (2) Is this relation symmetric? Explain your answer?

   (c) (2) Is this relation antisymmetric? Explain your answer?

   (d) (2) Is this relation transitive? Explain your answer?

   (e) (2) Is this relation a partial order? Explain your answer?

3. (6) Consider the relation $R = \{(a,b) \in \mathbb{N}^2 : |a-b| \leq 2\}$. For example $(1,3) \in R$ and $(3,1) \in R$ but $(1,4) \notin R$. Is R an equivalence relation? Explain your answer.

4. (6) Prove using mathematical induction that the sum of the first n natural numbers is less than or equal to n^2. This can also be stated as:

   Prove that the proposition P(n),

$$\sum_{i=1}^{n} i \leq n^2$$

   is true for all $n \in \mathbb{N}$.

5. (6) Show using the Division Algorithm theorem, that if $a|b$ and $a|c$ then $a|b+c$.

6. (6) Use mathematical induction to prove P(n), the proposition that $2|(n^2+n)$ for all $n \in \mathbb{N}$.