CISC-102 Fall 2016

Homework 10
Solutions

1. Pascal's triangle is symmetric about its central column. That is for an odd number of entries in a row (as in row 8) the same numbers are found when moving backward and forward from the central value 70. A row with an even number of entries such as row 5: 1 5 10 10 5 1, exhibits a similar pattern without a unique central value. Explain why Pascal's triangle exhibits this symmetry, using one of the binomial coefficient identities that we saw this week in class.

   The identity to use is

   $$\binom{n}{k} = \binom{n}{n-k}$$

   When n is odd we have

   $$\binom{n}{0} = \binom{n}{n}, \binom{n}{1} = \binom{n}{n-1}, \cdots, \binom{n}{\lfloor \frac{n}{2} \rfloor} = \binom{n}{\lceil \frac{n}{2} \rceil}$$

   And when n is even we have

   $$\binom{n}{0} = \binom{n}{n}, \binom{n}{1} = \binom{n}{n-1}, \cdots, \binom{n}{\frac{n}{2}-1} = \binom{n}{\frac{n}{2}+1}$$

2. Use a truth table to verify that the proposition $p \vee \neg(p \wedge q)$ is a tautology, that is, the expression is true for all values of p and q.

p	q	$p \wedge q$	$\neg(p \wedge q)$	$p \vee \neg(p \wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

3. Use a truth table to verify that the proposition $(p \wedge q) \wedge \neg(p \vee q)$ is a contradiction, that is, the expression is false for all values of p and q.

p	q	$p \wedge q$	$p \vee q$	$\neg(p \vee q)$	$(p \wedge q) \wedge \neg(p \vee q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

4. Use a truth table to show that $p \vee q \equiv \neg(\neg p \wedge \neg q)$.

p	q	$\neg p$	$\neg q$	$p \vee q$	$\neg p \wedge \neg q$	$\neg(\neg p \wedge \neg q)$
T	T	F	F	T	F	T
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	F	T	F

5. Show that the following argument is valid.

$$p \rightarrow q, \neg q \vdash \neg p$$

We need to show that $p \rightarrow q \wedge \neg q \rightarrow \neg p$ is a tautology, and we do so using a truth table as follows:

$\neg p$	p	q	$\neg q$	$p \rightarrow q$	$p \rightarrow q \wedge \neg q$	$p \rightarrow q \wedge \neg q \rightarrow \neg p$
F	T	F	T	F	F	T
F	T	T	F	T	F	T
T	F	T	F	T	F	T
T	F	F	T	T	T	T

6. Let A = {1,2,3,4,5}. Determine the truth value of each of the following statements.

   (a) $(\exists x \in A)(x + 2 = 7)$
   This is true with $x = 5$.

   (b) $(\forall x \in A)(x + 2 < 8)$
   This is true, because

   $$(1 + 2 < 8) \wedge (2 + 2 < 8) \wedge (3 + 3 < 8(\wedge(4 + 2 < 8) \wedge (5 + 2 < 8).$$

   (c) $(\exists x \in A)(x + 3 < 2)$
   This is false because:

   $$(1 + 3 \not< 2) \wedge (2 + 3 \not< 2) \wedge (3 + 3 \not< 2) \wedge (4 + 3 \not< 2) \wedge (5 + 3 \not< 2).$$

(d) $(\forall x \in A)(x + 3 \leq 9)$
This is true, because

$$(1 + 3 \leq 9) \wedge (2 + 3 \leq 9) \wedge (3 + 3 \leq 9) \wedge (4 + 3 \leq 9) \wedge (5 + 3 \leq 9).$$

7. Let A = {1,2,3,4,5}. And let $(x, y) \in A^2$, be the domain of the propositions given below. Determine the truth value of the following statements.

   (a) $\exists x \forall y, x^2 < y + 1$
   The statement is true because $1^2 < y + 1$ for every $y \in A$.

   (b) $\forall x \exists y, x^2 < y + 1$
   The statement is false because there is no $y \in A$ such that $5^2 < y + 1$.