CISC-102 FALL 2016

HOMEWORK 7 SOLUTIONS

Problems

(1) Prove that if $a \equiv b(\mod m)$ and $c \equiv d(\mod m)$ then $a - c \equiv b - d(\mod m)$.

Proof. $a \equiv b(\mod m)$ and $c \equiv d(\mod m)$ respectively imply:
$b - a = pm$ and $d - c = qm$ where p and q are integers.
Therefore we have:

$$\begin{aligned}(b-a)-(d-c) &= pm - qm \\ &= m(p-q)\end{aligned}$$

Therefore we can conclude that $m|(b-a)-(d-c)$ so $a - c \equiv b - d(\mod m)$.

□

(2) Write out each of the 5 residue classes mod 5 for integers in the range -10 to 10.

$[0]_5 = \{-10, -5, 0, 5, 10\}$

$[1]_5 = \{-9, -4, 1, 6\}$

$[2]_5 = \{-8, -3, 2, 7\}$

$[3]_5 = \{-7, -2, 3, 8\}$

$[4]_5 = \{-6, -1, 4, 9\}$

(3) How many different strings can you make using the letters TIMBITS?

There are 7 letters with 2 repeated pairs. Thus we have $\frac{7!}{2!2!}$ ways to arrange the letters.

(4) A store selling menswear has, 3 kinds of jackets, 7 kinds of shirts, and 5 kinds of pants. How many choices are there for a single item? How many choices are there for one of each kind of clothing item.

For selecting a single item we use the sum rule. We have three sets with pairwise empty intersections J, S, and P, such that $|J| = 3, |S| = 7,$ and $, |P| = 5$. The number of choices is the sum $|J| + |S| + |P| = 15$.

For selecting one of each we use the product rule. That is, $|J| \times |S| \times |P| = 105$.

(5) From 100 used cars siting on a lot, 20 are to be selected for a test designed to check safety requirements. These 20 cars will be returned to the lot, and again 20 will be selected for testing for emission standards.

   (a) In how many ways can the cars be selected for safety requirement testing?
   $\left(\begin{array}{c} 100 \\ 20 \end{array} \right)$

   (b) In how many ways can the cars be selected for emission standards testing?
   $\left(\begin{array}{c} 100 \\ 20 \end{array} \right)$

   (c) In how many different ways can the cars be selected for both tests?
   $\left(\begin{array}{c} 100 \\ 20 \end{array} \right) \left(\begin{array}{c} 100 \\ 20 \end{array} \right)$

   (d) In how many ways can the cars be selected for both tests if exactly 5 cars must be tested for safety and emission?
   $\left(\begin{array}{c} 100 \\ 5 \end{array} \right) \left(\begin{array}{c} 95 \\ 15 \end{array} \right) \left(\begin{array}{c} 80 \\ 15 \end{array} \right)$

(6) There are 230 students registered in CISC-102 this term. Prove that there is a month of the year in which at least 19 students in the class were born.

If we divide 230 by 12, the number of months of the year, we get 19.16. Now imagine twelve pigeon holes each containing 18 chairs. Each student in the class enters the pigeon hole representing their month of birth. But since the quotient of 230/12 is larger than 18, we know that there will be at least one pigeon hole where there are more people than chairs.

(7) How many binary strings of length 11 contain precisely 4 1s, and 7 0s? For example 00101010100 satisfies the requirement and so does 11110000000.

The number of such binary strings is:

$$\binom{11}{4} = \binom{11}{7}.$$

(8) You have 7 identical treats to distribute amongst 5 children.

(a) In how many different ways can the treats be distributed?
There are $\binom{11}{4}$ ways to do this.

(b) In how many different ways can the treats be distributed if each child must get at least one.
There are $\binom{6}{2}$ ways.