CISC-102 WINTER 2016

HOMEWORK 3

Please work on these problems and be prepared to share your solutions with classmates in class next Friday. Assignments will **not** be collected for grading.

Readings

Read sections 1.7 and 1.8 of *Schaum's Outline of Discrete Mathematics.*
Read section 2.1 of *Discrete Mathematics Elementary and Beyond.*

Problems

(1) Let $\{A_i : i \in \mathbb{N}\}$ denote an arbitrary indexed class of sets. Let $k \in \mathbb{N}$ Show that

$$\bigcap_{i \in \mathbb{N}} A_i \subseteq A_k \subseteq \bigcup_{i \in \mathbb{N}} A_i$$

(2) Prove using mathematical induction that the sum of the first n natural numbers is equal to $\frac{n(n+1)}{2}$. This can also be stated as:
Prove that the proposition $\mathrm{P}(n)$,

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

is true for all $n \in \mathbb{N}$

(3) Prove using mathematical induction that the proposition $\mathrm{P}(n)$,

$$\sum_{i=1}^{n} \frac{1}{2^i} = 1 - \frac{1}{2^n}$$

(4) Prove using mathematical induction that the proposition $\mathrm{P}(n)$, the number of values storable in a decimal string (a decimal string uses values, 0, 1, ..., 9) of length n is 10^n.

(5) Prove using mathematical induction that the proposition $\mathrm{P}(n)$, the number of values storable in a string using k different symbols of length n is k^n.