CISC-102 WINTER 2016

HOMEWORK 5

Please work on these problems and be prepared to share your solutions with classmates in class next Friday. Assignments will **not** be collected for grading.

Readings

Read sections 11.1, 11.2, 11.3, 11.4, and 11.5 of *Schaum's Outline of Discrete Mathematics.*

Read section 6.1, and 6.2 of *Discrete Mathematics Elementary and Beyond.*

Problems

(1) Evaluate
  (a) $|3-7|$
  (b) $|1-4|-|2-9|$
  (c) $|-6-2|-|2-6|$

(2) Find the quotient q and remainder r, as given by the Division Algorithm theorem for the following examples.
  (a) a = 500, b = 17
  (b) a = -500, b = 17
  (c) a = 500, b = -17
  (d) a = -500, b = -17

(3) Show that $c|0$, for all $c \in \mathbb{Z}, c \neq 0$.

(4) Let $a, b, c \in \mathbb{Z}$ such that $c|a$ and $c|b$. Let r be the remainder of the division of b by a, that is there is a $q \in \mathbb{Z}$ such that $b = qa + r, 0 \leq r \leq |b|$. Show that under these condition we have $c|r$.

(5) Let $a, b \in \mathbb{Z}$ such that $2|a$. (In other words a is even.) Show that $2|ab$.

(6) Let $a \in \mathbb{Z}$ show that $3|a(a+1)(a+2)$, that is the product of three consecutive integers is divisible by 3.

(7) Let a be any integer. Let $\mathrm{P}(n)$ denote the proposition:

$$\sum_{i=0}^{n} a^i = \frac{a^{n+1}-1}{a-1}$$

Prove that $\mathrm{P}(n)$ is true for all integers $n \geq 0$. Although the first form of induction would suffice to prove this result, use the second form of induction.