CISC-102 WINTER 2016

HOMEWORK 6

Please work on these problems and be prepared to share your solutions with classmates in class next Friday. Assignments will **not** be collected for grading.

Readings

Read sections 11.6 of *Schaum's Outline of Discrete Mathematics*.

Read section 6.6 (Don't worry if the theorems of this section seem daunting. The first 3 pages of the section do give a good explanation of gcd, and lcm.) of *Discrete Mathematics Elementary and Beyond*.

Problems

(1) Prove, using the second (strong) form of mathematical induction that any integer value greater than 2 can be written as $3a + 4b + 5c$, where a, b, c are non-negative integers, that is $a, b, c \in \mathbb{Z}, a, b, c \geq 0$. (HINT: You need to use 3 base cases, that is, verify that 3,4 and 5 can be written as $3a + 4b + 5c$, where a, b, c are non-negative integers.)

(2) Let a, b, c be Integers.
   (a) Prove that if $a|b$ and $b|c$ then $a|c$.
   (b) Prove that if $a|b$ and $a|c$, then $a|(b + c)$
   (c) Prove that if $a|b$ and $b|a$, then $|a| = |b|$, that is $a = \pm b$

(3) Let a = 1763, and b = 42
   (a) Find $\gcd(a, b)$. Show the steps used by Euclid's algorithm to find $\gcd(a, b)$.
   (b) Find integers x, y such that $\gcd(a, b) = ax + by$
   (c) Find lcm(a,b)

(4) Prove $\gcd(a, a + k)$ divides k.

(5) If a and b are relatively prime, that is $\gcd(a, b) = 1$ then we can always find integers x, y such that $1 = ax+by$. This fact will be useful to prove the following proposition.

   Suppose p is a prime such that $p|ab$, that is p divides the product ab, then $p|a$ or $p|b$.