CISC-102 WINTER 2016

HOMEWORK 9

Please work on these problems and be prepared to share your solutions with classmates in class next Friday. Assignments will **not** be collected for grading. Read sections 5.3 of *Schaum's Outline of Discrete Mathematics*.

Read section 3.1, 3.5 and 3.6 of *Discrete Mathematics Elementary and Beyond*.

Problems

(1) Consider the equation

$$\binom{12}{5} + \binom{12}{6} = \binom{13}{6}. \tag{1}$$

(a) Use a counting argument to prove that the left hand and right hand sides of equation (1) are in fact equal.

(b) Use algebraic manipulation to prove that the left hand and right hand sides of equation (1) are in fact equal.

(2) The equation

$$\binom{n-1}{k-1} + \binom{n-1}{k} = \binom{n}{k} \tag{2}$$

generalizes equation (1) above. Use algebraic manipulation to prove that the left hand and right hand sides of equation (2) are in fact equal. Note: A counting argument justifying the equation was given in class.

(3) In the notes for Lecture 16 you will find Pascal's triangle worked out for rows 0 to 8. The numbers in row 8 are 1 8 28 56 70 56 28 8 1. Work out the values of rows 9 and 10 of Pascal's triangle with the help of equation (2).

(4) Pascal's triangle is symmetric about its central column. That is for an odd number of entries in a row (as in row 8) the same numbers are found when moving backward and forward from the central value 70. A row with an even number of entries such as row 5: 1 5 10 10 5 1, exhibits a similar pattern without a unique central value. Explain why Pascal's triangle exhibits this symmetry by using one of the binomial coefficient identities that we saw this week in class.

(5) Prove that

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \cdots + (-1)^n \binom{n}{n} = 0$$

Note that this equation can also be written as follows:

$$\sum_{i=0}^{n} \binom{n}{i} (-1^i) = 0$$

HINT: This can be viewed as a special case of the binomial theorem.