

CISC-102

Fall 2015

Lecture 1

Introduction

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- Goodwin Hall Room 532
- Office Hours:
Wednesday 13:00- 15:00
- Course webpage:
<http://research.cs.queensu.ca/home/daver/102/2015Fall/>

Introduction

- Class Hours
Tuesday, 14:30-15:30
Wednesday, 16:30-17:30
Friday, 15:30-16:30
- Class Location: Chernoff Hall Auditorium

Introduction

- Homework every week. Keep up to date or you risk falling behind.
- Homework will be solved in class on due date.

Introduction

- 4 quizzes 17% each
- 1 final 32%

Motivation

- According to a recent poll (33,000 CDN students interviewed in 2014) the preferred employer is Google.
- I have been given a communication from an applicant to Google with tips on how to conduct an interview.

#1 tip Algorithm Complexity: You need to know Big-O. If you struggle with basic big-O complexity analysis, then you are almost guaranteed not to get hired.

Motivation

- Mastering Discrete Math is the direct prerequisite to mastering algorithms and complexity.
 - Send me suggestions for applications you would like to know more about (for example, Google search, encryption, computer gaming...) and I will explain how some topic we study is related to that application

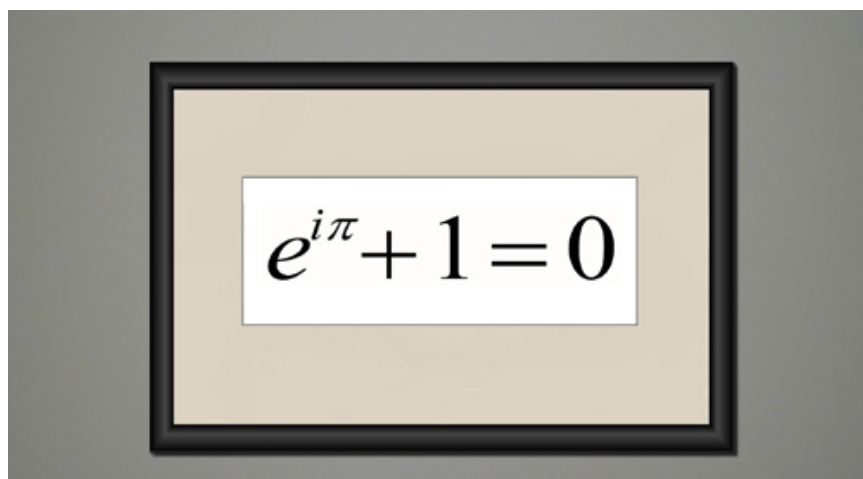
Motivation

- You should view this course as a language course. You will be learning the language of mathematics and computing!

Motivation

- Math can be fun.
- Math is beautiful!
- Math is a human invention just like music, painting, sculpture, poetry, hockey, basketball, soccer, fishing ...

Motivation

The image shows the mathematical equation $e^{i\pi} + 1 = 0$ centered within a white rectangular box. This box is set against a light beige background, which is itself enclosed in a dark grey frame. The entire composition is centered within a larger white area.

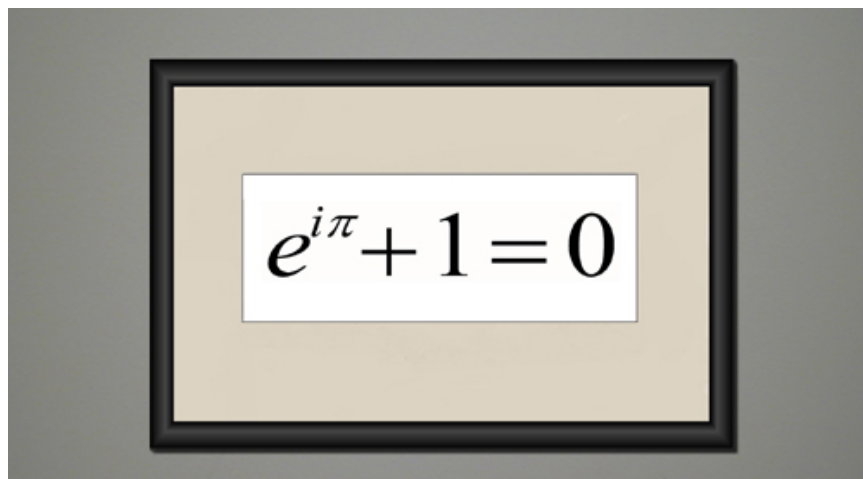
Motivation

- The picture on the previous slide is a work of art titled “beauty”. (Prints can be purchased on-line.)
- The equation $e^{i\pi} + 1 = 0$ consists of the most important numbers in mathematics 0, 1 (integers) π (an irrational real number) i (a complex number), operations + X and exponentiation and the relation =.

Math is a language

A mathematical joke asks,
 "How many mathematicians
 does it take to change a light
 bulb?"
 and answers " $-e^{i\pi}$ ".

Motivation

The image shows the equation $e^{i\pi} + 1 = 0$ centered within a white rectangular box. This box is surrounded by a thick black border, which is itself set against a light beige background. The entire composition is centered within a larger gray rectangular area.

Hand Shakes

- Alice is having a birthday party at her house, and has invited Bob, Carl, Diane, Eve, Frank, and George.
- They all shake hands with each other
- Q: How many handshakes?

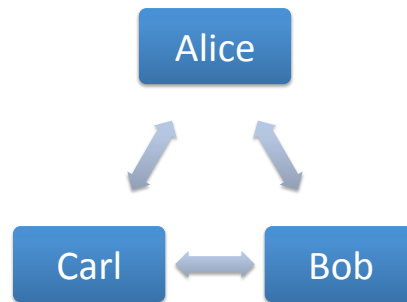
Hand Shakes

- How many handshakes?
 - Two people shaking hands with each other counts as one hand shake.
- Can you guess how many hand shakes with 7 people?
 - 7 people each shakes with 6 others. So $6*7 = 42$?
 - This leads to a formula for n people we would get $n*n-1$ hand shakes.
- How about 2 people?
 - Easy: 1 handshake. But $2*1 \neq 1$?

Hand Shakes

- No? How about 2 people?
 - Easy: 1 handshake
- 3 people? $3*2=6$. Correct answer is 3?
- It looks like the correct formula is $3*2/2$.

Hand Shakes



3×2 counts each handshake twice, so we divide by 2 to get the correct answer.

Hand Shakes

- How many handshakes?
 - Two people shaking hands with each other counts as one hand shake.
- Can you guess how many hand shakes with 7 people?
 - 7 people each shakes with 6 others.
 - So $7 \times 6 / 2 = 21$.
- In general we can say that for n people we have $n(n-1)/2$ hand shakes.

Hand Shakes

- The hand shake problem seems frivolous but it is actually a representation of an important mathematical concept.
- Computer Scientists need to study these concepts so that they can properly discuss the software that they develop (and use) in terms of correctness and efficiency.

Hand Shakes

- The hand shake problem seems frivolous but it is actually a representation of an important mathematical concept.
- For example if we wanted to know which handshake was the “best” we would have to compare $n(n-1)/2$ of them.
- Let $n = 35,000,000$ (the population of Canada) we would have to compare 612,499,982,500,000 or roughly 612 trillion hand shakes. (Too much!)

Hand Shakes

- Let $n = 35,000,000$ (the population of Canada) we would have to compare 612,499,982,500,000 or roughly 612 trillion hand shakes.
- If we test one handshake per second it would take roughly 31,688 Years, 269 Days, 1 Hour. (Too long!)

Hand Shakes

- Let's convert the hand shake problem into an "official" math problem using proper notation.
- The basic building block will be the set.

Sets

- Let's convert the hand shake problem into an "official" math problem using proper notation.
- The basic building block will be the set.

Examples:

$$A = \{1, 3, 5, 7, 9\}$$

$$B = \{x \mid x \text{ is an even integer, } x > 0\}$$

$$C = \{x : x \text{ is an odd integer, } x > 0\}$$

Sets

$$A = \{1, 3, 5, 7, 9\}$$

$$B = \{x \mid x \text{ is an even integer, } x > 0\}$$

$$C = \{x : x \text{ is an odd integer, } x > 0\}$$

examples

$A \subseteq C$ (A is a subset of C, or A is contained in C)

$C \supseteq A$ (C is a superset of A, or C contains A)

Sets

$\mathbf{N}$ = the set of *natural numbers*: $1, 2, 3, \dots$

$\mathbf{Z}$ = the set of all integers: $\dots, -2, -1, 0, 1, 2, \dots$

$\mathbf{Q}$ = the set of rational numbers

$\mathbf{R}$ = the set of real numbers

$\mathbf{C}$ = the set of complex numbers

Observe that $\mathbf{N} \subseteq \mathbf{Z} \subseteq \mathbf{Q} \subseteq \mathbf{R} \subseteq \mathbf{C}$.

Sets

$\mathbf{U}$: All sets under investigation in any application of set theory are assumed to belong to some fixed large set called the *universal set*.

$\emptyset$: A set with no elements is called the *empty set* or *null set* .

For any set A , we have: $\emptyset \subseteq A \subseteq \mathbf{U}$

Sets

Let $S = \{a, b, c, d, e, f, g\}$ denote the set of party goers.
A handshake can be represented as a two element subset of S , $\{a, b\}$.

Q. How many two element subsets are there of a set of n elements.

Sets

1.26 Which of the following sets are equal?

$$A = \{x \mid x^2 - 4x + 3 = 0\},$$

$$B = \{x \mid x^2 - 3x + 2 = 0\},$$

$$C = \{x \mid x \in \mathbf{N}, x < 3\},$$

$$D = \{x \mid x \in \mathbf{N}, x \text{ is odd}, x < 5\},$$

$$E = \{1, 2\},$$

$$F = \{1, 2, 1\},$$

$$G = \{3, 1\},$$

$$H = \{1, 1, 3\}.$$

Sets

1.26 Which of the following sets are equal?

$$A = \{x \mid x^2 - 4x + 3 = 0\}, A = \{x \mid (x-1)(x-3)=0\}, \mathbf{A}=\{1,3\}$$

$$B=\{x \mid x^2-3x+2=0\}, B = \{x \mid (x-1)(x-2) = 0\}, \mathbf{B}=\{1,2\}$$

$$C = \{x \mid x \in \mathbf{N}, x < 3\}, \mathbf{C} = \{1,2\}$$

$$D = \{x \mid x \in \mathbf{N}, x \text{ is odd}, x < 5\}, \mathbf{D} = \{1,3\}$$

$$E = \{1, 2\},$$

$$F = \{1,2,1\}, \mathbf{F} = \{1,2\} \text{ repetition does not change a set.}$$

$$G = \{3, 1\}, \text{ Order does not change a set.}$$

$$H = \{1,1,3\}. \mathbf{H} = \{1,3\}$$

Sets

1.1 Which of the following sets are equal?

$$\{x,y,z\}, \{z,y,z,x\}, \{y,x,y,z\}, \{y,z,x,y\}$$