CISC-102

Winter 2016

Lecture 2

Sets

Definition: (From Schaum's Notes)
A ***set*** may be viewed as any well-defined collection of objects, called the ***elements*** or *members* of the set.

This sentence defines in a mathematical sense the term *set* and the term *element.*
Key things to remember with sets.
Always use curly braces { } .

Sets represent an "unordered" collection of distinct objects.

$\boldsymbol{U}$ **:** All sets under investigation in any application of set theory are assumed to belong to some fixed large set called the *universal set.*

$\varnothing$ **:** A set with no elements is called the *empty set* or *null set* .

For any set A, we have: $\varnothing \subseteq A \subseteq \boldsymbol{U}$

The empty set is a subset of every set, and the universal set is a superset of every set.

Hand Shakes

Let's convert the hand shake problem into an "official" math problem using math notation.

The basic building block will be the set.

Sets

Let S = {a,b,c,d,e,f,g} denote the set of party goers.

A handshake can be represented as a two element subset of S, {a,b}.

Q. How many two element subsets are there of a set of n elements?

This question is a precise "model" of the handshake problem.

Notation

Consider the set $A = \{ 1, 2, 3\}$.

A is a set consisting of 3 elements.

$\{1\} \subseteq A$, ($\{1\}$ is a subset of A)

$1 \in A$ (1 is an element of A)

1.26 Which of the following sets are equal?

$A = \{x \mid x^2 - 4x + 3 = 0\}$,

$B = \{x \mid x^2 - 3x + 2 = 0\}$,

$C = \{x \mid x \in \mathbf{N}, x < 3\}$,

$D = \{x \mid x \in \mathbf{N}, x \text{ is odd}, x<5\}$,

$E = \{1, 2\}$,

$F = \{1,2,1\}$,

$G = \{3, 1\}$,

$H = \{1,1,3\}$.

1.1 Which of the following sets are equal?

$A = \{x,y,z\}$,

$B = \{z,y,z,x\}$,

$C = \{y,x,y,z\}$,

$D = \{y,z,x,y\}$.

Sets

Definition: (From Schaums)

A *set* may be viewed as any well-defined collection of objects, called the *elements* or *members* of the set.

In this context "well-defined" means distinct or distinguishable objects.

Examples:

Set of people in this room right now.

Set of coins in my pocket.

Note: If I have two (or more) quarters then I need to be able to distinguish one from the other if I want to consider the coins as a set. If I have no coins in my pocket then the set of coins in my pockets is an *empty* set.

More notation:
Subsets, supersets
$A \subseteq C$ (A is a subset of C, or A is contained in C)
$C \supseteq A$ (C is a superset of A, or C contains A)

Proper subsets, proper supersets
$A \subset C$ (A is a proper subset of C, or A is contained in C, and not equal to C.)
$C \supset A$ (C is a proper superset of A, or C contains A, and is not equal to A.)

Seating Arrangements

There is a large table at the party and Alice wants to experience every possible seating arrangement. How many ways can 7 people sit at a table?

This “seating arrangement question” is equivalent to asking for the number of different ways to order 7 people.

Number of ways to order 1 person? 1.
Number of ways to order 2 people? (1,2) (2,1). 2*1
Number of ways to order 3 people? (3,1,2) (3,2,1)
(1,3,2)(2,3,1)(1,2,3)(2,13). 3*2*1
Number of ways to order 4 people?
Guess: 4*3*2*1= 4!

Permutations

There are *n!* ways to order n distinct objects

Recall $n!$ (n factorial) is given by the expression:

$$n! = n\times (n-1) \times(n-2) \times \ldots \times 1$$

Lottery Tickets

Lotto 6-49, players choose 6 numbers from 1 to 49.
How many ways are there to choose to these numbers?

A very simplified version of this game is Lotto 1-49, where players choose 1 number from 49. There are 49 choices.

Note that the probability (the odds) of winning Lotto 1-49 is 1/49. (one choice divided by the total number of choices)

Consider Lotto 2-49, where you have to pick 2 numbers from 49.
A tempting but wrong guess would be 49×48 choices.

Suppose choice 1 is 42, and choice 2 is 18. That is equivalent to choice 1 is 18 and choice 2 is 42, so 49×48 double counts all possibilities. The actual answer is $49 \times 48/2!$.

Combinations

For lotto 6-49, players choose 6 numbers from 1 to 49.

How many ways are there to choose to these numbers?

Solution:
$49 \times 48 \times 47 \times 46 \times 45 \times 44 / 6! = 13{,}983{,}816$.

This can also be written as:

$$\binom{49}{6} = \frac{49!}{43!6!}$$

and pronounced 49 choose 6.

What is the probability that any single choice is the winning number?

1/13,983,816

The current price of a Lotto 6-49 card is $3.

The “fair” prize for choosing the winning numbers should be $41,951,448.

The actual prize this week (Jan. 6, 2016) is $5,000,000.