CISC-102
Winter 2016
Lecture 3

- Homework every week.
- Not handed in no grades.
- Quizzes based on homework.
- Friday class: Homework day.
- Go over current week's homework.
  (Answer your questions)
- Go over prior week's solutions.

Readings

Read sections 1.1, 1.2, 1.3, and 1.4 of *Schaum's Outline of Discrete Mathematics*.
Read sections 1.1, 1.2 and 1.3 of *Discrete Mathematics Elementary and Beyond*.

The readings for week 1 and homework 1.

Seating Arrangements

There is a large table at the party and Alice wants to experience every possible seating arrangement. How many ways can 7 people sit at a table?

This “seating arrangement question” is equivalent to asking for the number of different ways to order 7 people.

Number of ways to order 1 person? 1.
Number of ways to order 2 people? (1,2) (2,1).
2*1
Number of ways to order 3 people? (3,1,2)
(3,2,1)
(1,3,2)(2,3,1)(1,2,3)(2,13). 3*2*1
Number of ways to order 4 people?
Guess: 4*3*2*1= 4!

Permutations

There are *n!* ways to order n distinct objects

Recall $n!$ (n factorial) is given by the expression:

$$n! = n \times (n-1) \times (n-2) \times \ldots \times 1$$

Lottery Tickets

Lotto 6-49, players choose 6 numbers from 1 to 49.
How many ways are there to choose to these numbers?

A very simplified version of this game is Lotto 1-49, where players choose 1 number from 49. There are 49 choices.

Note that the probability (the odds) of winning Lotto 1-49 is 1/49. (one choice divided by the total number of choices)

Consider Lotto 2-49, where you have to pick 2 numbers from 49.
A tempting but wrong guess would be 49×48 choices.

Suppose choice 1 is 42, and choice 2 is 18. That is equivalent to choice 1 is 18 and choice 2 is 42, so 49×48 double counts all possibilities. The actual answer is $49 \times 48/2!$.

Combinations

For lotto 6-49, players choose 6 numbers from 1 to 49.

How many ways are there to choose to these numbers?

Solution:
$49\times48\times47\times46\times45\times44/6! = 13{,}983{,}816$.

This can also be written as:

$$\binom{49}{6} = \frac{49!}{43!6!}$$

and pronounced 49 choose 6.

What is the probability that any single choice is the winning number?

1/13,983,816

The current price of a Lotto 6-49 card is $3.

The “fair” prize for choosing the winning numbers should be $41,951,448.

The actual prize this week (Jan. 6, 2016) is $5,000,000.

Set Operators

Operators on sets are “union” and “intersection”.

Definitions:

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

Logical Operators

$p \wedge q$ pronounced p and q

Both p and q have to be true for the compound proposition p and q to be true.

$p \vee q$ pronounced p or q

At least one of p or q must be true for the compound proposition p or q to be true.

We can rewrite our definition for set union and set intersection using logical operators as follows:

$$A \cup B = \{x : x \in A \vee x \in B\}$$

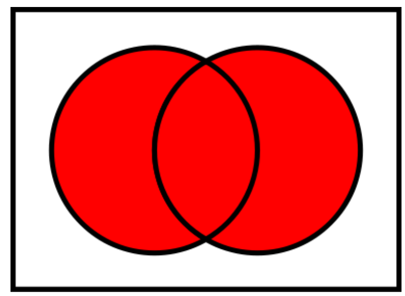
$$A \cap B = \{x : x \in A \wedge x \in B\}$$

Venn Diagrams

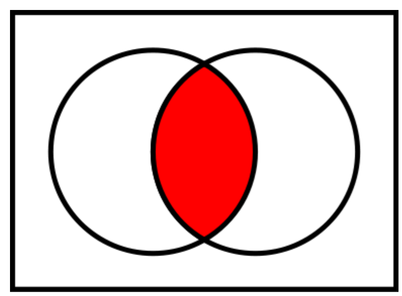
Useful for providing intuitive insight.

Note the rectangle surrounding the circles denotes the Universe **U**.

$$A \cup B = \{x : x \in A \vee x \in B\}$$



$$A \cap B = \{x : x \in A \wedge x \in B\}$$



For example:

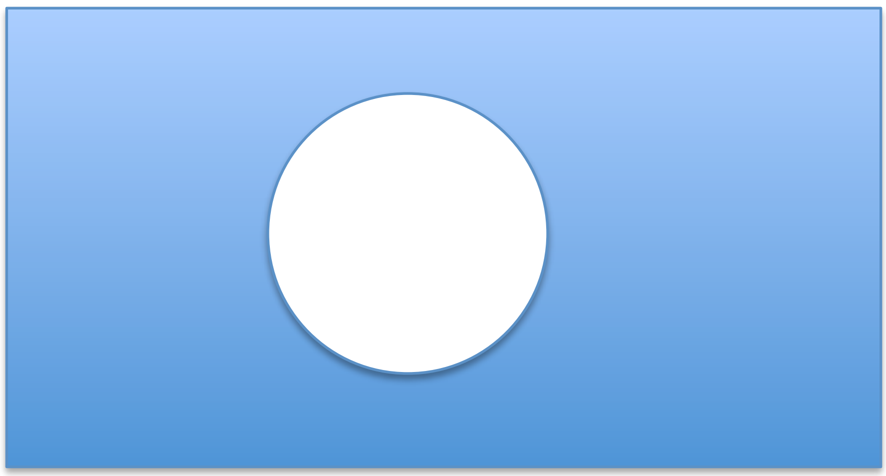
Suppose A is the set of guitars and B is the set of red musical instruments.

- An element x is in the set of A union B if it is a guitar or if it is a red musical instrument.

- An element of x is in the set of A intersection B if x is red and x is a guitar.

The complement of a set A written A^c is defined as:

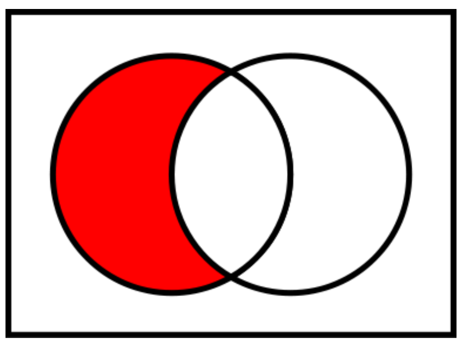
$$A^c = \{x | x \notin A\}$$



The relative complement of a set B with respect to A, sometimes called the difference is:

$$A \backslash B = \{x \mid x \in A, x \notin B\}$$

(The relative complement is sometimes written as A – B.)

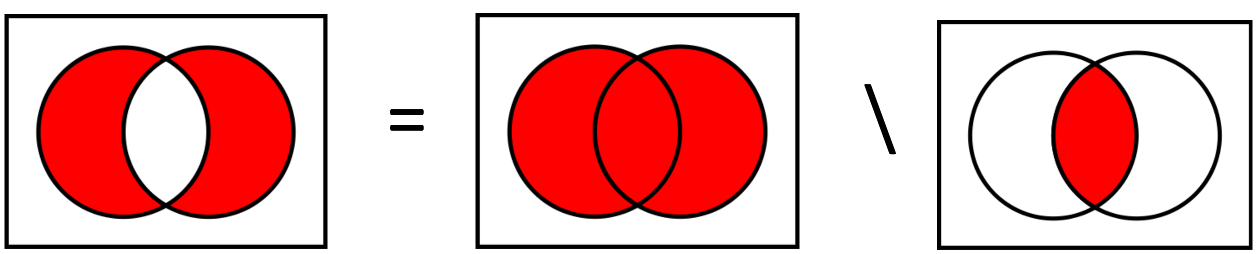


The symmetric difference of sets A and B:

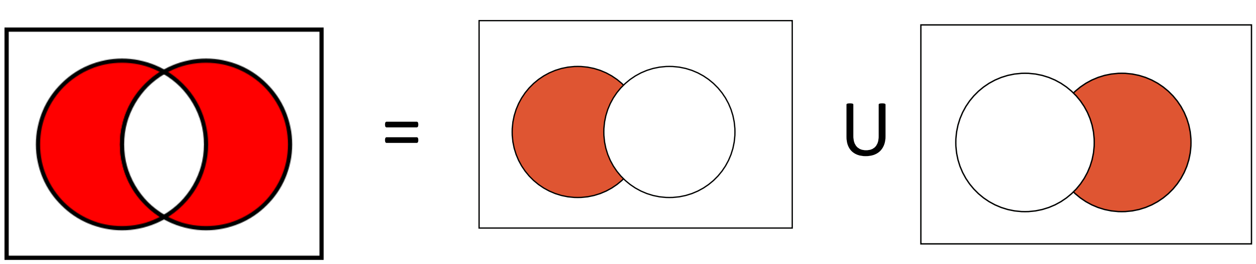
$$A \oplus B = (A \cup B) \backslash (A \cap B) \quad \text{or} \quad A \oplus B = (A \backslash B) \cup (B \backslash A)$$

The symmetric difference consists of elements that are in A or in B but not in both.

$$A \oplus B = (A \cup B) \backslash (A \cap B)$$



$$A \oplus B = (A \backslash B) \cup (B \backslash A)$$



Problems

1.4 Let $\mathbf{U} = \{1, 2, \ldots, 9\}$ be the universal set, and let

$$\begin{array}{lll} A = \{1, 2, 3, 4, 5\}, & C = \{5, 6, 7, 8, 9\}, & E = \{2, 4, 6, 8\}, \\ B = \{4, 5, 6, 7\}, & D = \{1, 3, 5, 7, 9\}, & F = \{1, 5, 9\}. \end{array}$$

Find: (*a*) $A \cup B$ and $A \cap B$; (*b*) $A \cup C$ and $A \cap C$; (*c*) $D \cup F$ and $D \cap F$.

(*a*) $A^{C}, B^{C}, D^{C}, E^{C}$; (*b*) $A \backslash B, B \backslash A, D \backslash E$; (*c*) $A \oplus B,\ C \oplus D,\ E \oplus F$.