# CISC-102 <br> Winter 2016 <br> Lecture 4 

The first quiz will be held on Tuesday, January 26.
1.4 Let $\mathbf{U}=\{1,2, \ldots, 9\}$ be the universal set, and let

$$
\begin{array}{lll}
A=\{1,2,3,4,5\}, & C=\{5,6,7,8,9\}, & E=\{2,4,6,8\}, \\
B=\{4,5,6,7\}, & D=\{1,3,5,7,9\}, & F=\{1,5,9\} .
\end{array}
$$

Find: (a) $A \cup B$ and $A \cap B$; (b) $A \cup C$ and $A \cap C$; (c) $D \cup F$ and $D \cap F$.
(a) $A^{\mathrm{C}}, B^{\mathrm{C}}, D^{\mathrm{C}}, E^{\mathrm{C}} ; \quad$ (b) $A \backslash B, B \backslash A, D \backslash E ; \quad$ (c) $A \oplus B, C \oplus D, E \oplus F$.

How many ways are there to pick 7 people out of a class of 70 and seat them into 7 numbered chairs? (Selection with ordering.)

1st pick has 70 choices.
2nd pick has 69 choices.
3rd pick has 68 choices.
4th pick has 67 choices.
5th pick has 66 choices.
6th pick has 65 choices.
7th pick has 64 choices.
So the number of ways to pick 7 people out of a class of 70 is:
$70 \times 69 \times 68 \times 67 \times 66 \times 65 \times 64=70!/ 63!$

# Ticket to paradise: Powerball jackpot could hit \$1.3B <br> (USA TODAY Jan. 11, 2016) 

Powerball Lottery game: Choose (un-ordered choice) 5 numbers from 69, (white balls), then a 6th number from 26 (red balls).

The number of choices are:

$$
\begin{aligned}
& \binom{69}{5} \times\binom{ 26}{1}=\frac{69!}{5!64!} \times \frac{26!}{25!1!} \\
& =292,201,338
\end{aligned}
$$

The cost of a ticket is $\$ 2$ so with an investment of $\$ 584,402,676$ you would be a guaranteed winner! However, there are additional details see: http://www.pennlive.com/news/2016/01/ how to win_powerball_you_can_g.html

| Idempotent laws: | (1b) $A \cap A=A$ |
| :--- | :--- |
| Associative laws: | $(2 \mathrm{~b})(A \cap B) \cap C=A \cap(B \cap C)$ |
| Commutative laws: | $(3 \mathrm{~b}) A \cap B=B \cap A$ |
| Distributive laws: | $(4 \mathrm{~b}) A \cap(B \cup C)=(A \cap B) \cup(A \cap C)$ |
| Identity laws: | $(5 \mathrm{~b}) A \cap \mathbf{U}=A$ |
|  | $(6 \mathrm{~b}) A \cap \emptyset=\emptyset$ |

Verify these properties with $\mathbf{U}=\{1,2,3,4,5,6,7\}$, $A=\{1,2,3\}, B=\{2,4,6\} C=\{4,5\}$.

| Idempotent laws: | (1a) $A \cup A=A$ |
| :--- | :--- |
| Associative laws: | (2a) $(A \cup B) \cup C=A \cup(B \cup C)$ |
| Commutative laws: | (3a) $A \cup B=B \cup A$ |
| Distributive laws: | (4a) $A \cup(B \cap C)=(A \cup B) \cap(A \cup C)$ |
| Identity laws: | (5a) $A \cup \emptyset=A$ |
|  | (6a) $A \cup \mathbf{U}=\mathbf{U}$ |

Verify these properties with $\mathrm{U}=\{1,2,3,4,5,6,7\}$, $A=\{1,2,3\}, B=\{2,4,6\}$.

| Involution laws: | (7) $\left(A^{\mathrm{C}}\right)^{\mathrm{C}}=A$ |
| :---: | :---: |
| Complement laws: | (8a) $A \cup A^{\mathrm{C}}=\mathbf{U}$ |
|  | (9a) $\mathbf{U}^{\mathrm{C}}=\emptyset$ |
| DeMorgan's laws: | (10a) $(A \cup B)^{\mathrm{C}}=A^{\mathrm{C}} \cap B^{\mathrm{C}}$ |
| Verify these properties with $\mathrm{U}=\{1,2,3,4,5,6,7\}$, |  |


| Complement laws: | $(8 \mathrm{~b}) A \cap A^{\mathrm{C}}=\emptyset$ |
| :--- | :--- |
|  | $(9 \mathrm{~b}) \emptyset^{\mathrm{C}}=\mathbf{U}$ |
| DeMorgan's laws: | $(10 \mathrm{~b})(A \cap B)^{\mathrm{C}}=A^{\mathrm{C}} \cup B^{\mathrm{C}}$ |

Verify these properties with $\mathrm{U}=\{1,2,3,4,5,6,7\}$,
$\mathrm{A}=\{1,2,3\}, \mathrm{B}=\{2,4,6\}$.

Definition: A set $S$ is said to be finite if $S$ is empty or if $S$ contains exactly $m$ elements where $m$ is a positive integer; otherwise $S$ is infinite.

Some finite sets:
$\mathrm{A}=\{1,2,3,4\}$,
$B=\{x: x \in \mathbb{N}, x \geq 0, x \leq 1\}$
Some infinite sets:
$C=\{1,2,3,4, \ldots\}$,
$\mathrm{D}=\{\mathrm{x}: \mathrm{x} \in \mathbb{R}, \mathrm{x} \geq 0, \mathrm{x} \leq 1\}$
Notation: We use vertical bars || to denote the size or cardinality of a finite set.
$|\mathrm{A}|=4$.
$B=\{1\}(0$ is not in $\mathbb{N})$, so $|B|=1$.

# Definition: The set of all (different) subsets of S 

 is the power set of S , which we denote as $\mathrm{P}(\mathrm{S})$.If $S$ is a finite set we can prove that:
$|\mathrm{P}(\mathrm{S})|=2^{|\mathrm{S}|}$.
Some examples:
$\varnothing$ : the empty set has 0 elements, and 1 subset. So $|P(\varnothing)|=2^{0}$.
$\{\mathrm{a}\}$ : has 1 element and 2 subsets.
So $|\mathrm{P}(\{\mathrm{a}\})|=2^{1}$.
$\{\mathrm{a}, \mathrm{b}\}$ : has 2 elements and 4 subsets.
So $|\mathrm{P}(\{\mathrm{a}, \mathrm{b}\})|=2^{2}$.

