CISC-102 Winter 2016

Quiz 2

February 9, 2016

Student ID: Solutions

Read the questions carefully. Please clearly state any assumptions that you make that are not explicitly stated in the question. Please answer all questions in the space provided. Use the back of pages for scratch work. There are 4 pages to this quiz. Note that (x) denotes the question is worth x points.

1. Suppose that a relation R is a partial order.

   (a) (1) Is R reflexive?

   Yes.

   (b) (1) Is R symmetric?

   No.

   (c) (1) Is R antisymmetric?

   Yes.

   (d) (1) Is R transitive?

   Yes.

2. (4) Consider the relation $R_2 = \{(a, b) \in \mathbb{N}^2 : |a - b| = 2\}$. For example $(1, 3) \in R_2$ and $(3, 1) \in R_2$ but $(1, 4) \notin R_2$. Is R_2 an equivalence relation? Explain your answer.

An equivalence relation is reflexive, symmetric, & transitive

R_2 is not transitive because $(1,3) \in R_2$ & $(3,5) \in R_2$ but $(1,5) \notin R_2$.

So R_2 is not an equivalence relation. R_2 is not reflexive but it is symmetric.

3. (6) Let $n \in \mathbb{N}$ and P(n) be the proposition:

$$\sum_{i=1}^{n} 2i - 1 = 1 + 3 + 5 + \cdots + 2n - 1 = n^2.$$

Use mathematical induction to prove that P(n) is true for all $n \in \mathbb{N}$.

Base: $n=1$, $2-1=1=1^2$

Ind. Hyp. $P(k)$ is true, that is

$$\sum_{i=1}^{k} 2i-1 = k^2$$

Ind. Step. (Goal: $\sum_{i=1}^{k+1} 2i-1 = (k+1)^2$)

$$\sum_{i=1}^{k+1} 2i-1 = \sum_{i=1}^{k}(2i-1) + 2k+1$$

$$= k^2 + 2k + 1$$

$$= (k+1)^2 \quad \blacksquare$$

4. (6) Let P(n) be the proposition

$$\sum_{i=1}^{n} \frac{1}{i \times (i+1)} = \frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \cdots + \frac{1}{n \times (n+1)} = \frac{n}{n+1}$$

Use mathematical induction to prove that P(n) is true for all $n \in \mathbb{N}$.

Base: $n=1$ $\quad \frac{1}{1 \times 2} = \frac{1}{2} = \frac{1}{1+1}$

Ind. Hyp. $P(k)$ is true, that is,

$$\sum_{i=1}^{k} \frac{1}{i(i+1)} = \frac{k}{k+1}$$

Ind. Step. $\left(\text{Goal } \sum_{i=1}^{k+1} \frac{1}{i(i+1)} = \frac{k+1}{k+2} \right)$

$$\sum_{i=1}^{k+1} \frac{1}{i(i+1)} = \sum_{i=1}^{k} \left(\frac{1}{i(i+1)} \right) + \frac{1}{(k+1)(k+2)}$$

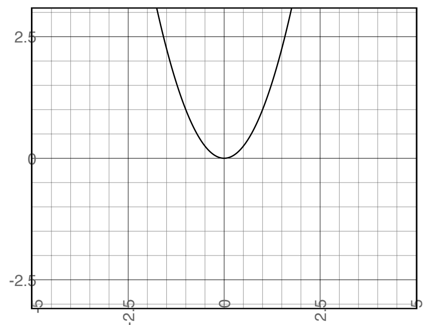
$$= \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$$

$$= \frac{k(k+2)+1}{(k+1)(k+2)}$$

$$= \frac{k^2+2k+1}{(k+1)(k+2)}$$

$$= \frac{(k+1)(k+1)}{(k+1)(k+2)} = \frac{k+1}{k+2} \quad \blacksquare$$

5. Consider the function $f : \mathbb{R} \to \mathbb{R}$ such that $f(x) = x^2$, as plotted below.



(a) (1) Is f a one-to-one function? Explain why or why not?

No $f(-x) = f(x)$

(b) (1) Is f an onto function? Explain why or why not?

No f is never negative

(c) (1) Is f a bijective function? Explain why or why not?

NO

6. Let $\mathbb{R}^+$ be defined as the set $\mathbb{R}^+ = \{x : x \in \mathbb{R}, x \geq 0\}$. This is the set of positive real numbers and zero, also known as the set of non-negative real numbers. Consider the function $g : \mathbb{R} \to \mathbb{R}^+$ such that $g(x) = x^2$. The is the same function as in the previous question with a different codomain.

(a) (1) Is g a one-to-one function? Explain why or why not?

No $f(-x) = f(x)$

(b) (1) Is g an onto function? Explain why or why not?

Yes Every element of $\mathbb{R}^+$ is an image of f.

(c) (1) Is g a bijective function? Explain why or why not?

No