# CISC-102 Winter 2016

Quiz 3

March 8, 2016

Student ID: Solutions

Read the questions carefully. Please clearly state any assumptions that you make that are not explicitly stated in the question. Please answer all questions in the space provided. Use the back of pages for scratch work. There are 3 pages to this quiz. Note that ( x ) denotes the question is worth x points.

1. Let a = 51, and b = 33.

   (a) ( 4 ) Find g = gcd(a,b). Show the steps used by Euclid's algorithm to find gcd(a,b).

   $51 = 33 + 18$

   $33 = 18 + 15$

   $18 = 15 + 3$

   $15 = (5)3 + 0$

   $\therefore \gcd(51, 33) = 3$

   (b) ( 2 ) Find lcm(a,b), and show the method used to find it.

   $$\text{lcm}(a,b) = \frac{a \times b}{\gcd(a,b)} = \frac{51 \times 33}{3}$$

   $$= 51 \times 11 = 561$$

2. ( 4 ) Prove $\gcd(a, a+k)$ divides $k$ for $a, k \in \mathbb{N}$.

Let $g = \gcd(a, a+k)$. Thus $g | a$ & $g | (a+k)$.
Also $g | ((a+k) - a)$ so $g | k$.

3. ( 2 ) Prove that $\gcd(a, a+2) = 1$ or $2$ for $a \in \mathbb{N}$.

From question 2 we can deduce that
$g | 2$. The only numbers that divide 2
are 1 & 2.

4. ( 4 ) We learned that by definition $a \equiv b (\text{mod } m)$ requires that $m|(a-b)$. Prove that $a \equiv b (\text{mod } m)$ implies that there is an integer $k$ such that $a = km + b$

$m | (a-b)$ implies that there is an
integer $k$ such that $(a-b) = km$.
Adding $b$ to the right & left of the equation
we get $a = km + b$

5. ( 4 ) In how many ways can the letters POPCORN be rearranged?

$$\frac{7!}{2!\,2!}$$

6. ( 6 ) Use the second form of induction, that is, strong induction, to prove that any integer value greater than or equal to 2 can be written as $2a + 3b$, where $a, b$ are non-negative integers, that is $a, b \in \mathbb{Z}, a, b \geq 0$. (Hint: Consider separate cases.)

Base! $2 = 2(1) + 3(0)$
$3 = 2(0) + 3(1)$

Ind. hyp. All numbers $j, 2 \leq j \leq k$ can be written as $2a + 3b$.

Ind. Step. Conside the intege $k+1$. By the Ind. hyp. we have $k = 2a + 3b$. There are two cases to consider.

case 1. $a > 0$.
$k + 1 = 2(a-1) + 3(b+1)$

case 2 $a = 0$
$k+1 = 2(a+2) + 3(b-1)$ $\boxtimes$