CISC-102
Winter 2016

Quiz 4

March 29, 2016

Student ID: Solutions

Read the questions carefully. Please clearly state any assumptions that you make that are not explicitly stated in the question.

Please answer all questions in the space provided. Use the back of pages for scratch work. NO CALCULATORS. There are 4 pages to this quiz. Note that (x) denotes the question is worth x points.

1. (6) Complete the following truth table.

p	q	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$
F	F	T	T	T
F	T	T	F	F
T	F	F	T	F
T	T	T	T	T

2. (6) Show that the proposition $\neg(p \wedge q)$ is logically equivalent to $\neg p \vee \neg q$, that is, show that

$$\neg(p \wedge q) \equiv \neg p \vee \neg q.$$

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

3. (6) Prove the validity of the equation given below. You may use algebra, or a counting argument. But please provide only one answer.

$$\binom{11}{7} + \binom{11}{6} = \binom{12}{7}.$$

$$\frac{11!}{7!\,4!} + \frac{11!}{6!\,5!} = \frac{(5)11! + (7)11!}{7!\,5!} = \frac{11!\,(12)}{7!\,5!}$$

$$= \frac{12!}{7!\,5!} = \binom{12}{7}$$

OR But not both

The number of ways to select 7 from 12 (without ordering) is equal to selecting 7 from 11 ensuring that a favourite is selected plus 6 from 11 ensuring that a favourite is not selected.

4. The binomial theorem can be stated as

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k.$$

(a) (4) Expand the expression $(x+y)^5$ using the binomial theorem.

$$\binom{5}{0}x^0y^5 + \binom{5}{1}x^1y^4 + \binom{5}{2}x^2y^3$$
$$+ \binom{5}{3}x^3y^2 + \binom{5}{4}x^4y^1 + \binom{5}{5}x^5y^0$$
$$= y^5 + 5xy^4 + 10x^2y^3 + 10x^3y^2 + 5x^4y + x^5$$

(b) (4) Use the binomial theorem to prove that

$$\sum_{k=0}^{4} \binom{4}{k} = 2^4.$$

$$(1+1)^4 = \sum_{k=0}^{4} \binom{4}{k} (1^{4-k})(1^k)$$

5. (6) You have 4 different math books, 3 different computing books and 3 different psychology books that you store on the same shelf. You like to keep things organized and keep books of the same subject together (in any order). In how many different ways can you arrange the books?

$3!\ 4!\ 3!\ 3!$

6. (6)Four people meet one night to play cards. Each starts with exactly k dollars. Bets must always be full dollar amounts. If a player loses all of her money then she must stop playing. Thus, the outcome at the end of the night can have each of the four players with 0 up to $4 \times k$ dollars. How many different outcomes can there be for this night of card playing?

$$\frac{(4K+3)!}{(4K)!\ 3!} = \binom{4K+3}{3}$$