CISC-102 WINTER 2016

HOMEWORK 1 SOLUTIONS

Problems

(1) Rewrite the following statements using set notation, and then give an example by listing members of sets that match the description. For example: A is a subset of C. <u>Answer:</u> $A \subseteq C$. $A = \{1, 2\}$, $C = \{1, 2, 3\}$.

There are many different solutions to these questions. I have shown several possibilities.

(a) The element 1 is not a member of (the set) A.
$1 \notin$ A. A $= \{2, 4\}$.

(b) The element 5 is a member of B.
$5 \in$ B. B $= \{5,6\}$.

(c) A is not a subset of D.
A $\not\subseteq$ D. A $= \{2, 4\}$ and D $= \{42, 18\}$.

(d) E and F contain the same elements.
E = F. E = F $= \{7\}$. E $\subseteq$ F and F $\subseteq$ E.

(e) A is the set of integers larger than three and less than 12.
A $= \{\ x : x \in \mathbb{Z}, 3 < x < 12\ \}$. A $= \{$ 4, 5, 6, 7, 8, 9, 10, 11$\}$.

(f) B is the set of even natural numbers less than 15.
B $= \{\ 2x : x \in \mathbb{N}, x < 8\}$. B $= \{2,4,6,8,10,12,14\}$. Some books define the naturals as including 0, and 0 is a multiple of 2 so it's even. Then B $= \{0,2,4,6,8,10,12,14\}$. However, for the rest of this course we will assume that $0 \notin \mathbb{N}$.

(g) C is the set of natural numbers x such that $4 + x = 3$.
C $= \{x : x \in \mathbb{N}, 4 + x = 3\ \}$. C $= \emptyset$.

(2) $A = \{x : 3x = 6\}$. $A = 2$, true or false? A $= \{2\}$. A $\neq 2$, so the statement is false.

(3) Which of the following sets are equal $\{r, s, t\}$, $\{t, s, r\}$, $\{s, r, t\}$, $\{t, r, s\}$. They are all equal. The order in which elements are written in a set is not important, unless ellipses "$\ldots$" are used to denote a sequence. For example $x = \{1, 2, \ldots, 10\}$.

(4) Consider the sets $\{4, 2\}$, $\{x : x^2 - 6x + 8 = 0\}$, $\{x : x \in \mathbb{N}, x \text{ is even}, 1 < x < 5\}$. Which one of these sets is equal to $\{4, 2\}$?

They are all equal.

(5) Which of the following sets are equal: $\emptyset$, $\{\emptyset\}$, $\{0\}$. None are equal. $\{\emptyset\}$ is a set within a set. 0 is a number not a set, and definitely not the empty set.

(6) Explain the difference between $A \subseteq B$, and $A \subset B$, and give example sets that satisfy the two statements.

$A \subseteq B$ is pronounced as "A is a subset of B" implying that A is a subset of B that may also be equal to A. $A = B = \{1\}$ and $A \subseteq B$. $A \subset B$ is pronounced "A is a proper subset of B " implying that A is strictly a subset of B. $A = \{1\}, B = \{1, 2\}$ and $A \subset B$.

(7) Consider the following sets $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 4, 5, 6, 7\}$, $C = \{3, 4\}$, $D = \{4, 5, 6\}$, $E = \{3\}$.

(a) Let X be a set such that $X \subseteq A$ and $X \subseteq B$. Which of the sets A, B, C, D, E could be X?
X could be the set $C = \{3, 4\}$ or the set $E = \{3\}$.

(b) Let $X \not\subseteq D$ and $X \not\subseteq B$.Which of the the sets A, B, C, D, E above could be X?
Set A is the only set from the list that is not a subset of D and not a subset of B.

(c) Find the smallest set M that contains all five sets.
M = {1,2,3,4,5,6,7}

(d) Find the largest set N that is a subset of all five sets. $N = \emptyset$

(8) Is an "element of a set", a special case of a "subset of a set"?

No, an element of a set is not a subset.

(9) List all of the subsets of the set $\{1, 2, 3\}$.

$\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$.

(10) List all of the subsets of the set $\{2, 3\}$.

$\emptyset, \{2\}, \{3\}, \{2, 3\}$.

(11) List all of the subsets of the set $\{1, 2, 3\}$ containing 1.

$\{1\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3\}$.

(12) Let $A = \{1, 2, 3, 4\}$. List all the subsets of A containing 1 but not containing 4.

$\{1\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3\}$.

(13) Consider the sets A ={1,2,3,4,5,6},B = {1,2,3,4},C = {5}, $D = \{6\}$, $E = \{1, 2\}$, $F = \{2, 3\}$, $G = \{3, 4\}$, and U is the set of Natural numbers the universe for this collection of sets. Draw a Venn diagram representing this collection of sets.