

# CISC-102 Winter 2016

## Homework 10 Solutions

1. Use a truth table to verify that the proposition  $p \vee \neg(p \wedge q)$  is a tautology, that is, the expression is true for all values of  $p$  and  $q$ .

$p$	$q$	$p \wedge q$	$\neg(p \wedge q)$	$p \vee \neg(p \wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

2. Use a truth table to verify that the proposition  $(p \wedge q) \wedge \neg(p \vee q)$  is a contradiction, that is, the expression is false for all values of  $p$  and  $q$ .

$p$	$q$	$p \wedge q$	$p \vee q$	$\neg(p \vee q)$	$(p \wedge q) \wedge \neg(p \vee q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

3. Use a truth table to show that  $p \vee q \equiv \neg(\neg p \wedge \neg q)$ .

$p$	$q$	$\neg p$	$\neg q$	$p \vee q$	$\neg p \wedge \neg q$	$\neg(\neg p \wedge \neg q)$
T	T	F	F	T	F	T
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	F	T	F

4. Show that the following argument is valid.

$$p \rightarrow q, \neg q \vdash \neg p$$

We need to show that  $p \rightarrow q \wedge \neg q \rightarrow \neg p$  is a tautology, and we do so using a truth table as follows:

$\neg p$	$p$	$q$	$\neg q$	$p \rightarrow q$	$p \rightarrow q \wedge \neg q$	$p \rightarrow q \wedge \neg q \rightarrow \neg p$
F	T	F	T	F	F	T
F	T	T	F	T	F	T
T	F	T	F	T	F	T
T	F	F	T	T	T	T

5. Let  $A = \{1,2,3,4,5\}$ . Determine the truth value of each of the following statements.

(a)  $(\exists x \in A)(x + 2 = 7)$

This is true with  $x = 5$ .

(b)  $(\forall x \in A)(x + 2 < 8)$

This is true, because

$$(1 + 2 < 8) \wedge (2 + 2 < 8) \wedge (3 + 2 < 8) \wedge (4 + 2 < 8) \wedge (5 + 2 < 8).$$

(c)  $(\exists x \in A)(x + 3 < 2)$

This is false because:

$$(1 + 3 \not< 2) \wedge (2 + 3 \not< 2) \wedge (3 + 3 \not< 2) \wedge (4 + 3 \not< 2) \wedge (5 + 3 \not< 2).$$

(d)  $(\forall x \in A)(x + 3 \leq 9)$

This is true, because

$$(1 + 3 \leq 9) \wedge (2 + 3 \leq 9) \wedge (3 + 3 \leq 9) \wedge (4 + 3 \leq 9) \wedge (5 + 3 \leq 9).$$

6. Let  $A = \{1,2,3,4,5\}$ . And let  $(x, y) \in A^2$ , be the domain of the propositions given below. Determine the truth value of the following statements.

(a)  $\exists x \forall y, x^2 < y + 1$

The statement is true because  $1^2 < y + 1$  for every  $y \in A$ .

(b)  $\forall x \exists y, x^2 < y + 1$

The statement is false because there is no  $y \in A$  such that  $5^2 < y + 1$ .