CISC-102 WINTER 2016

HOMEWORK 4 SOLUTIONS

Problems

(1) Let $A = \{1, 2, 3\}$ and $B = \{a, b\}$.

(a) What is $A \times B$?

$$A \times B = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}.$$

(b) What is $B \times A$?

$$B \times A = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}.$$

(c) What is $(A \times B) \cup (B \times A)$?
$(A \times B) \cup (B \times A) = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b),$
$(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}.$

(d) What is $(A \times B) \cap (B \times A)$?

$$(A \times B) \cap (B \times A) = \emptyset.$$

(2) Suppose A is a set of m elements, and B is a set of n elements. How many elements are there in the product set $A \times B$? How many elements are there in the product set $B \times A$?

$$|A \times B| = |B \times A| = |A| \times |B| = m \times n.$$

(3) Consider the following relations on the set $A = \{1, 2, 3\}$:

- $R = \{(1, 1), (1, 2), (1, 3), (3, 3)\}$,
- $S = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$,

- $T = \{(1,1),(1,2),(2,2),(2,3)\}$,
- $A \times A$.

Which of the relations above are antisymmetric?

R and T are antisymmetric.

(4) Explain why each of the following binary relations on the set $S = \{1,2,3\}$ is or is not an equivalence relation on S.

An equivalence relation is a relation that is reflexive, symmetric, and transitive.

(a) $R = \{(1,1),(1,2),(3,2),(3,3),(2,3),(2,1)\}$
Not reflexive { because (2,2) is missing }, also not transitive {because (1,3) is missing }.

(b) $R = \{(1,1),(2,2),(3,3),(2,1),(1,2),(3,2),(2,3),(3,1),(1,3)\}$
This is an equivalence relation.

(c) $R = \{(1,1),(2,2),(3,3),(3,1),(1,3)\}$
This is an equivalence relation.

(5) Let R be a relation on the set of Natural numbers such that $(a,b) \in$ R if $a \geq b$. Show that the relation R on $\mathbb{N}$ is a partial order.

A relation is a partial order if it is reflexive, antisymmetric, and transitive.

R is reflexive because $a \geq a$ for all natural numbers a.

R is antisymmetric because whenever $a \geq b$ and $b \geq a$ we have $a = b$.

R is transitive because whenever $a \geq b$ and $b \geq c$ we have $a \geq c$.

(6) Determine whether the mappings from $\mathbb{R}$ to $\mathbb{R}$ shown below are or are not functions, and explain your decision.

(a) $f(x) = 1/x$.
$f(0)$ is undefined so $f(x)$ is not a function.

(b) $f(x) = \sqrt{x}$.
$f(x)$ is undefined if x is negative so $f(x)$ is not a function. Also if $\sqrt{x}$ denotes positive and negative roots we don't have a unique image for positive real numbers.

(c) $f(x) = 3x - 3$.
$f(x)$ is uniquely defined for all $x \in \mathbb{R}$. So $f(x)$ is a function.

(7) Determine whether each of the following functions from $\mathbb{R}$ to $\mathbb{R}$ is a bijection, and explain your decision. HINT: Plotting these functions may help you with your decision.

(a) $f(x) = 3x + 4$
$f(x)$ is one-to-one and onto so it is a bijection.

(b) $f(x) = -x^2 + 2$
$f(x)$ is not one-to-one because $f(-a) = f(a)$ for any $a \in \mathbb{R}$. Furthermore $f(x)$ is not onto because there is no $x \in \mathbb{R}$ with an image that is greater than 2.

(c) $f(x) = x^3 - x^2$
$f(x)$ is not one-to-one because $f(x) = 0$ for $x = 0$ and $x = 1$.

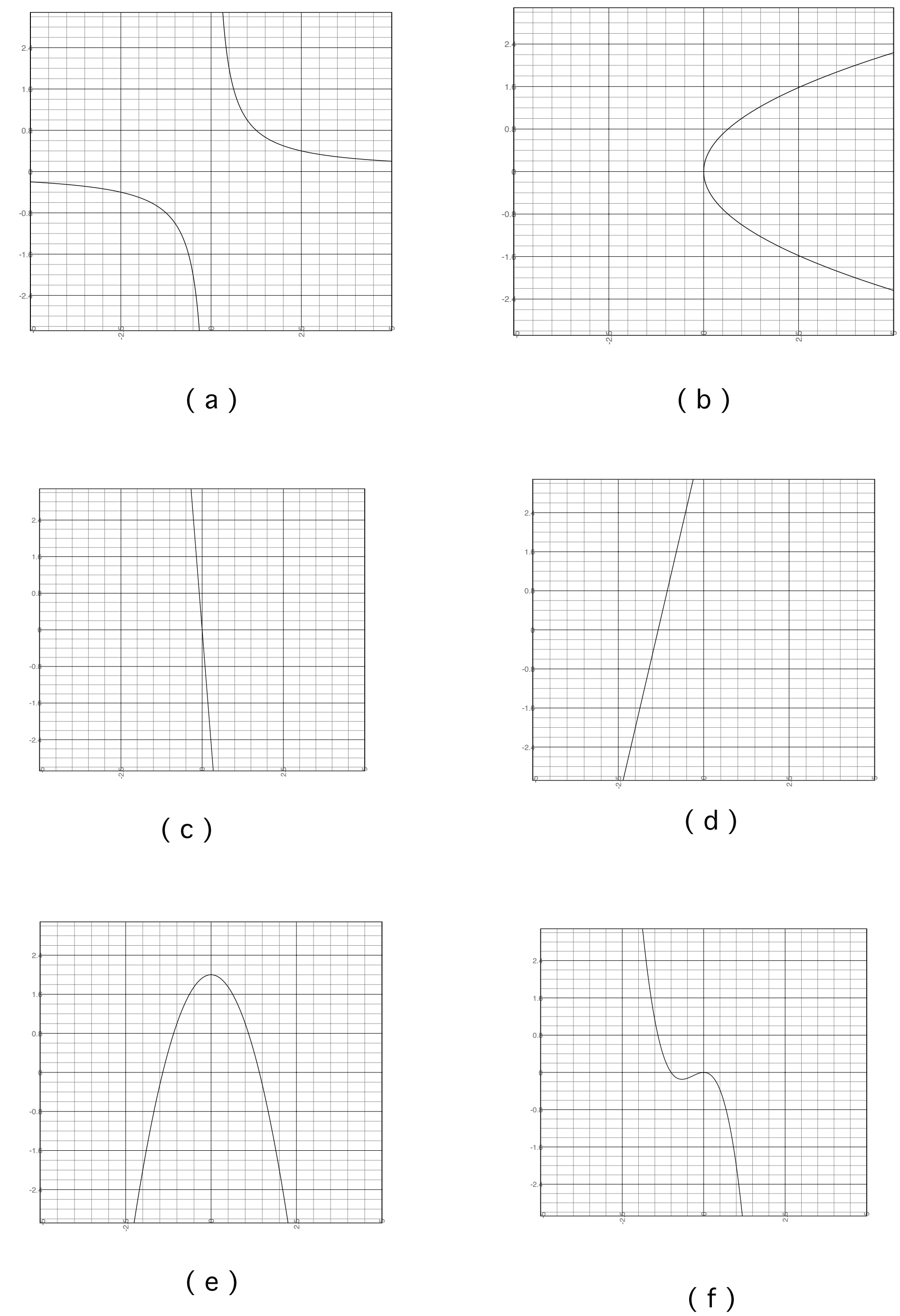

FIGURE 1. (a) $1/x$ (b) $\sqrt{x}$ (c) $3x-3$ (d) $3x+4$ (e) $-x^2+2$ (f) x^3-x^2