CISC-102 WINTER 2016

HOMEWORK 7 SOLUTIONS

(1) Let a $= 2^2 * 3 * 7^2$ and b $= 2 * 3^2 * 7$

(a) Find g = gcd(a,b). Show how the prime factorization is helpful for finding gcd(a,b).

Given the prime factorization of a and b we can use the formula:

$$\gcd(a, b) = 2^{\min(2,1)} 3^{\min(1,2)} 7^{\min(1,2)} = 2 * 3 * 7$$

(b) Find lcm(a,b). Show how the prime factorization is helpful for finding lcm(a,b).

Given the prime factorization of a and b we can use the formula:

$$\text{lcm}(a, b) = 2^{\max(2,1)} 3^{\max(1,2)} 7^{\max(1,2)} = 2^2 * 3^2 * 7^2$$

(2) Let a, b, m be positive integers, such that $a \equiv b(\mod m)$. Show that this implies that there exists an integer q so that $a = b + qm$. For example: Suppose $a = 4, b = 8$

and $m = 2$. Then this implies that there is an integer q such that $4 = 8 + q2$. In this example it is easy to see that $q = -2$.

If $a \equiv (b \mod m)$ then by definition we have $m|(a-b)$.

Therefore, there exists an integer q such that:

$$a - b = qm. \tag{1}$$

Adding b to the left and right sides of equation (1) yields the desired result, that is: $a = b + qm$. $\square$

(3) A store selling menswear has, 3 kinds of jackets, 7 kinds of shirts, and 5 kinds of pants. How many choices are there for a single item? How many choices are there for one of each kind of clothing item.

There are $3+7+5 = 15$ choices for a single item. There are $3 \times 7 \times 5 = 105$ choices for one of each kind.

(4) Suppose we have 9 signal flags that are hung on a vertical flag pole, such that there are 4 identical red flags, 2 identical blue flags, and 3 identical green flags. How many different signals can be made using all 9 flags.

There are $\frac{9!}{4!2!3!} = 1260$ different signals.

(5) How many different strings can you make using the letters TIMBITS?

There are $\frac{7!}{2!2!}$ different strings.

(6) A restaurant has 6 different deserts on the menu. How many ways are there to choose 1 desert? 2 different deserts? 3 different deserts?

There are $\binom{6}{1} = 6$ ways to choose 1 desert.

There are $\binom{6}{2} = 15$ ways to choose 2 different deserts.

There are $\binom{6}{3} = 20$ ways to choose 3 different deserts.