CISC-102 FALL 2017

HOMEWORK 2

Please work on these problems and be prepared to share your solutions with classmates in class next Monday. Assignments will **not** be collected for grading.

Readings

Read sections 1.5, 1.6, 1.7, 1.8 of *Schaum's Outline of Discrete Mathematics*.

Read sections 1.1, 1.2 and 1.3 again (if you did not understand things last week) of *Discrete Mathematics Elementary and Beyond*.

Problems

(1) Illustrate DeMorgan's Law $(A \cap B)^c = A^c \cup B^c$ using Venn diagrams.

(2) Let $A_i = \{1, 2, 3, \ldots, i\}$ for all $i \in \mathbb{N}$. For example $A_4 = \{1, 2, 3, 4\}$.
   What are the elements of the set:
   (a) What are the elements of the set $\cup_{i=1}^{n} A_i$?
   (b) What are the elements of the set $\cap_{i=1}^{n} A_i$?

(3) Observe that $A \subseteq B$ has the same meaning as $A \cap B = A$. Draw a Venn diagram to illustrate this fact.

(4) Use a Venn diagram to show that if $A \subseteq B$ **and** $B \subseteq C$, then $A \subseteq C$.

(5) Use the Principle of Exclusion and Inclusion to show that $|A \cup B| + |A \cap B| = |A| + |B|$. (It may help your understanding if you first explore an example such as A = {1,2,3} and B = {3,4})

(6) What are the cardinalities of the following sets?
   (a) A = {winter, spring, summer, fall}.
   (b) B = $\{x : x \in \mathbb{Z}, 0 < x < 7\}$.
   (c) $\mathcal{P}(B)$, that is, the power set of B.
   (d) C = $\{ x : x \in \mathbb{N}, x \text{ is even} \}$

(7) Suppose that we have a sample of 100 students at Queen's who take at least one of the following language courses, French-101, Spanish-101, German-101. Also suppose that 65 take French-101, 45 take German-101, 42 take Spanish 101, 20 take French-101 and German-101, 25 take French-101and Spanish-101, and 15 take German-101 and Spanish-101.
   (a) How many students take all three language courses? (HINT: Use the Principle of Inclusion and Exclusion to write an expression representing these students and the classes they take.)
   (b) Draw a Venn diagram representing these 100 students and fill in the regions with the correct number.

(c) How many students take exactly 1 of these courses?
(d) How many students take exactly 2 of these courses?

(8) Let S={a,b,c,d,e,f,g}. Determine which of the following are partitions of S:
(a) P1 = [{a,c,e},{b},{d,g}]
(b) P2 = [{a,b,e,g},{c},{d,f}]
(c) P3 =[{a,e,g},{c,d},{b,e,f}]
(d) P4= [{a,b,c,d,e,f,g}]

(9) Recall that the union operation is associative, that is $A\cup(B\cup C) = (A\cup B)\cup C$. Show that the relative complement set operation is not associative, that is, $A\backslash(B\backslash C) = (A\backslash B)\backslash C$, is incorrect for some sets A, B, C. (Note if relative complement is associative then the equation must be true for all sets A, B, C.)

(10) Consider a set S of n elements, such that $\{a, b\} \subseteq S$.
(a) What is the cardinality of the power set of $S \setminus \{a\}$?
(b) What is the cardinality of the power set of $S \setminus \{a, b\}$?
(c) How many subsets of S are there that contain the element a?
(d) How many subsets of S are there that contain the element a and exclude the element b?