CISC-102 FALL 2017

HOMEWORK 5

Please work on these problems and be prepared to share your solutions with classmates in class next week. Assignments will **not** be collected for grading.

Readings

Read sections 2.1, 2.2, 2.3, 11.1, 11.2, 11.3, 11.4, and 11.5 of *Schaum's Outline of Discrete Mathematics*.

Read section 6.1, and 6.2 of *Discrete Mathematics Elementary and Beyond*.

Problems

(1) Consider the following relations on the set $A = \{1, 2, 3\}$:
- $R = \{(1, 1), (1, 2), (1, 3), (3, 3)\}$,
- $S = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$,
- $T = \{(1, 1), (1, 2), (2, 2), (2, 3)\}$,
- $A \times A$.

For each of these relations determine whether it is symmetric, antisymmetric, reflexive, or transitive.

(2) Explain why each of the following binary relations on the set $S = \{1, 2, 3\}$ is or is not an equivalence relation on S.
  (a) $R = \{(1, 1), (1, 2), (3, 2), (3, 3), (2, 3), (2, 1)\}$
  (b) $R = \{(1, 1), (2, 2), (3, 3), (2, 1), (1, 2), (3, 2), (2, 3), (3, 1), (1, 3)\}$
  (c) $R = \{(1, 1), (2, 2), (3, 3), (3, 1), (1, 3)\}$

(3) Let R be a relation on the set of Natural numbers such that $(a, b) \in$ R if $a \geq b$. Show that the relation R on $\mathbb{N}$ is a partial order.

(4) Evaluate
  (a) $|3 - 7|$
  (b) $|1 - 4| - |2 - 9|$
  (c) $|-6 - 2| - |2 - 6|$

(5) Find the quotient q and remainder r, as given by the Division Algorithm theorem for the following examples.
  (a) a = 500, b = 17
  (b) a = -500, b = 17
  (c) a = 500, b = -17
  (d) a = -500, b = -17

(6) Show that $c|0$, for all $c \in \mathbb{Z}, c \neq 0$.

(7) Let $a, b, c \in \mathbb{Z}$ such that $c|a$ and $c|b$. Let r be the remainder of the division of b by a, that is there is a $q \in \mathbb{Z}$ such that $b = qa + r, 0 \leq r < |b|$. Show that under these condition we have $c|r$.

(8) Let $a, b \in \mathbb{Z}$ such that $2|a$. (In other words a is even.) Show that $2|ab$.

(9) Let $a \in \mathbb{Z}$ show that $3|a(a+1)(a+2)$, that is the product of three consecutive integers is divisible by 3.

(10) Use induction to prove the following propositions.
   (a) $n^3 + 2n$ is divisible by 3, for all $n \in \mathbb{N}, n \geq 1$.
   (b) Show that any integer value greater than 2 can be written as $3a + 4b + 5c$, where a, b, c are non-negative integers, that is $a, b, c \in \mathbb{Z}, a, b, c \geq 0$.
   (c) Show that every Natural number n can be represented as a sum of distinct powers of 2. For example the number $42 = 32 + 8 + 2 = 2^5 + 2^3 + 2^1$.