

CISC-102 Fall 2017

A

Quiz 1

October 4 , 2017

Solutions

Student ID: _____

Read the questions carefully. Please clearly state any assumptions that you make that are not explicitly stated in the question.

Please answer all questions in the space provided. Use the back of pages for scratch work. There are ?? pages to this quiz. Note that (x) denotes the question is worth x points.

CALCULATORS ARE NOT PERMITTED.

For the following multiple choice questions underline the one answer that you think is correct.

For example: What is the smallest positive integer?

1. 42

2. -1

3. 1

4. 0

5. \emptyset

1. (2) If $P = \{a, b, c, d, e\}$, $Q = \{a, c, e, d, t\}$ and $R = \{t, d, c, b, e\}$, then the intersection of P, Q and R , $P \cap Q \cap R$ is:

(a) $\{a, c\}$

(b) $\{a, c, e\}$

(c) $\{c, d\}$

(d) $\{c, d, e\}$

(e) \emptyset

2. (2) If set $A = \{2, 9, 5\}$, then what is $|P(A)|$, the cardinality of the powerset of A , or in plain English the number of subsets which can be formed from A ?

(a) 3

(b) 9

(c) 5

(d) 8

(e) 16

3. (2) The cardinality of set $A = \{5, 6, 3, 2, 3, 2\}$, $|A|$ is:

(a) 6

(b) 5

(c) 4

(d) 3

(e) 16

4. (2) If the universal set is $U = \{2, 4, 6, 8, 10\}$ and $F = \{4, 10\}$, then the complement of F , F^c is:

(a) $\{1, 3, 5, 7, 9\}$

(b) $\{2, 6, 8\}$

(c) $\{4, 6, 8, 10\}$

(d) $\{5, 7, 9\}$

(e) $\{2, 4, 6, 8, 10\}$

5. (2) If $H = \{a, d, c, v\}$, and $J = \{r, c, f\}$ then $|(H \cup J)| =$
- (a) 5
 - (b) $\{a, c, d, r, f, v, t, w\}$
 - (c) $\{a, c, d, r, f, v\}$
 - (d) 6
 - (e) 1
6. (2) If $H = \{a, d, c, v\}$, $J = \{r, c, f\}$ then $H \cap J$ is:
- (a) $\{a, d, r, f, v, t\}$
 - (b) $\{a, d, v\}$
 - (c) $\{c\}$
 - (d) $\{a, d, v, t\}$
 - (e) \emptyset
7. (2) Suppose there are 20 journalists covering an international event. 17 of the journalists speak English, 15 speak French, and 18 speak Spanish. Furthermore suppose that 12 speak French and English, 15 speak English and Spanish, and 13 speak French and Spanish. How many speak all three languages?
- (a) 0
 - (b) 10
 - (c) 12
 - (d) 15
 - (e) 18
8. (2) Suppose there are 20 journalists covering an international event. 17 of the journalists speak English, 15 speak French, and 18 speak Spanish. (As in the previous question.) Without any further information what is the maximum number of journalists that speak all three languages?
- (a) 0
 - (b) 10
 - (c) 12
 - (d) 15
 - (e) 18

9. Consider the proposition $P(n)$

$$\sum_{i=4}^n i = \frac{(n-3)(n+4)}{2}$$

Complete a proof by induction that $P(n)$ is true for all natural numbers $n \geq 4$ by answering the following questions.

(a) (2) What is the base case?

Base: $n=4$ $\frac{(4-3)(4+4)}{2} = 4$

(b) (2) What is the induction hypothesis?

Ind. Hyp. Assume $\sum_{i=4}^k i = \frac{(k-3)(k+4)}{2}$, for $k \in \mathbb{N}$, $k \geq 4$.

(c) (2) What is the induction step?

$$\begin{aligned} \sum_{i=4}^{k+1} i &= \sum_{i=4}^k i + k+1 \\ &= \frac{(k-3)(k+4)}{2} + k+1 \end{aligned}$$

$$= \frac{k^2 + k - 12 + 2k + 2}{2}$$

$$= \frac{k^2 + 3k - 10}{2}$$

$$= \frac{(k-2)(k+5)}{2}$$

\square