CISC-102 Fall 2017

Quiz 2 B

October 25, 2017

Student ID: Solutions

Read the questions carefully. Please clearly state any assumptions that you make that are not explicitly stated in the question.

Please answer all questions in the space provided. Use the back of pages for scratch work. There are 5 pages to this quiz. Note that (x) denotes the question is worth x points.

CALCULATORS ARE NOT PERMITTED.

For the following multiple choice questions underline the one answer that you think is correct.

For example: What is the smallest positive integer?

1. 42
2. -1
3. <u>1</u>
4. 0
5. $\emptyset$

1. (2) Consider the relation on the natural numbers, $R_1 = \{(a, b) \in \mathbb{N}^2 : a \leq b\}$

   (a) R_1 is reflexive, symmetric, antisymmetric, and transitive.

   (b) R_1 is reflexive, NOT symmetric, antisymmetric, and transitive.

   (c) R_1 is NOT reflexive, symmetric, NOT antisymmetric, and NOT transitive.

   (d) R_1 is reflexive, symmetric, NOT antisymmetric, and NOT transitive.

   (e) R_1 is NOT reflexive, symmetric, antisymmetric, and transitive.

2. (2) Consider the relation on the set $A = \{1, 2, 3, 4\}$: $R_2 = \{(1, 1), (2, 2), (4, 4)\}$,

   (a) R_2 is reflexive, symmetric, antisymmetric, and transitive.

   (b) R_2 is reflexive, NOT symmetric, antisymmetric, and transitive.

   (c) R_2 is NOT reflexive, symmetric, NOT antisymmetric, and NOT transitive.

   (d) R_2 is reflexive, symmetric, NOT antisymmetric, and NOT transitive.

   (e) R_2 is NOT reflexive, symmetric, antisymmetric, and transitive.

3. (2) Consider the relation $R_3 = \{(a, b) \in \mathbb{N}^2 : |a - b| \leq 2\}$. For example $(1, 3) \in R_3$ and $(3, 1) \in R_3$ but $(1, 4) \notin R_3$.

   (a) R_3 is reflexive, symmetric, antisymmetric, and transitive.

   (b) R_3 is reflexive, symmetric, NOT antisymmetric, and NOT transitive.

   (c) R_3 is reflexive, NOT symmetric, antisymmetric, and transitive.

   (d) R_3 is NOT reflexive, symmetric, NOT antisymmetric, and NOT transitive.

   (e) R_3 is NOT reflexive, symmetric, antisymmetric, and transitive.

4. (2) Consider a relation R_4 that is a partial order.

   (a) R_4 is reflexive, symmetric, antisymmetric, and transitive.

   (b) R_4 is reflexive, symmetric, antisymmetric, and NOT transitive.

   (c) R_4 is reflexive, NOT symmetric, antisymmetric, and transitive.

   (d) R_4 is NOT reflexive, symmetric, NOT antisymmetric, and NOT transitive.

   (e) R_4 is NOT reflexive, symmetric, antisymmetric, and transitive.

5. (2) Let $\mathbb{R}^+$ denote the positive real numbers. Consider the function $f : \mathbb{R} \to \mathbb{R}^+$ such that $f(x) = 2^x$, as plotted below.

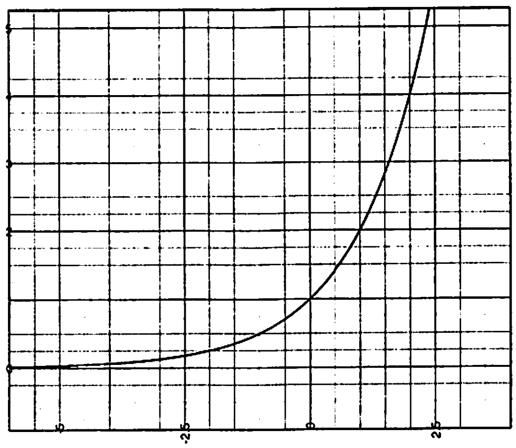

Figure 1: $f(x) = 2^x$

(a) f a one-to-one function, NOT an onto function

(b) f is both a one-to-one and onto function

(c) f is neither one-to-one nor onto

(d) f is not an injective function

(e) f is not a bijective function

6. (4) Let $a, b \in \mathbb{N}$, such that $a|b$ and $b|a$. Underline every statement below that is true. (Note there may be more than one true statement, or no true statements.)

(a) There exists a $q \in \mathbb{N}$ such that $a = bq$

(b) There exists a $p \in \mathbb{N}$ such that $b = ap$

(c) $a = apq$

(d) $pq = 1$

(e) $a = b$

7. Consider the recursively defined function $f(1) = 1, f(n) = 2f(n-1) + 1$, for all natural numbers $n, n \geq 2$.

   (a) (2) What are the values of $f(2), f(3), f(4), f(5)$?

   $f(2) = 3 \quad f(3) = 7 \quad f(4) = 15 \quad f(5) = 31$

   (b) (6) Prove that $f(n) = 2^n - 1$ using mathematical induction.

   Base: $f(1) = 2^1 - 1 = 1$

   Ind. hyp.: Assume that $f(k) = 2^k - 1$ for some fixed value $k \in \mathbb{N}$, $k \geq 1$.

   Ind. Step: $f(k+1) = 2f(k) + 1$ (by def f)

   $= 2(2^k - 1) + 1$ (ind. hyp.)

   $= 2^{k+1} - 2 + 1$

   $= 2^{k+1} - 1$

   Therefore by the principle of mathematical induction $f(n) = 2^n - 1$ for all natural numbers n. $\boxtimes$

8. (6)Prove using the 2nd form of mathematical induction, that every natural number $n, n > 1$, is either prime or a product of primes.

Base: 2 is prime

Ind Hyp: Assume that j is either prime or a product of primes for all j. $2 \le j \le K$.

Ind Step: Consider $K+1$. If $K+1$ is prime, we are done. Otherwise $K+1$ must be composite, so we can write $K+1 = ab$, such that $a, b \in \mathbb{N}$ and $2 \le a \le K$, $2 \le b \le K$. Now apply the induction hypothesis to both a & b to obtain a product of primes that is equal to $K+1$.

Therefore, by the principle of mathematical induction we have shown that every natural number n, $n > 1$ is either prime or a product of primes.