

CISC-102 Fall 2017

Quiz 3 B

November 8, 2017

Student ID: Solutions

Read the questions carefully. Please clearly state any assumptions that you make that are not explicitly stated in the question. Please answer all questions in the space provided. Use the back of pages for scratch work. There are ?? pages to this quiz. Note that (x) denotes the question is worth x points.

CALCULATORS ARE NOT PERMITTED.

Note: There is no need to simplify your expressions.

1. (4) Underline all of the statements below that are true.

(a) $5 \equiv 5 \pmod{5}$

(b) $5 \equiv 10 \pmod{20}$

(c) $20 \equiv 5 \pmod{10}$

(d) $10 \equiv 20 \pmod{5}$

(e) $20 \equiv 10 \pmod{5}$

2. (4) In how many different ways can the letters KINGSTON be rearranged? Underline each correct answer.

(a) $8!$

(b) $\frac{8!}{2!}$

(c) $\frac{8!}{2!6!}$

(d) $\binom{8}{2}$

(e) $7!$

3. (4) Consider the greatest common divisor of 45 and 10, $\gcd(45, 10)$. Underline all of the statements below that are true.

(a) $\gcd(45, 10) = \gcd(10, 4)$

(b) $\gcd(45, 10) = \gcd(5, 0)$

(c) There exists integers a, b such that $\gcd(45, 10) = 45a + 10b$

(d) $\gcd(45, 10) = 1$

(e) 45 and 10 are relatively prime.

4. (4) Consider the least common multiple of 33 and 6, $\text{lcm}(33, 6)$. Underline all of the statements below that are true.

(a) $\text{lcm}(33, 6) = \gcd(33, 6) - 33 \times 6$

(b) $\text{lcm}(33, 6) = \frac{\gcd(3, 0)}{33 \times 6}$

(c) $\text{lcm}(33, 6) \times \gcd(33, 6) = 33 \times 6$

(d) $\text{lcm}(33, 6) + \gcd(33, 6) = 33 \times 6$

(e) $\text{lcm}(33, 6) = \gcd(33, 6)$

5. (4) Consider a bag containing 10 balls numbered from 1 to 10. In how many ways can 5 balls be selected, without ordering, and without replacement? Underline each correct answer.

(a) $5!$

(b) $10!$

(c) $\frac{10!}{5!5!}$

(d) $\binom{10}{2}$

(e) $\frac{10!}{5!}$

6. (4) Consider a bag containing 10 balls numbered from 1 to 10. In how many ways can 5 balls be selected, without ordering, and without replacement, so that all 5 numbers are even or all 5 are odd? Explain how you arrived at your answer.

There is only one even selection $\{2, 4, 6, 8, 10\}$
 and one odd selection $\{1, 3, 5, 7, 9\}$
 so the total number of selections is 2.

7. (4) Write out the 4 residue classes (mod 4) for integers in the range -10 to 10.

$$\begin{aligned} [0]_4 &= \{-8, -4, 0, 4, 8\} \\ [1]_4 &= \{-7, -3, 1, 5, 9\} \\ [2]_4 &= \{-10, -6, -2, 2, 6, 10\} \\ [3]_4 &= \{-9, -5, -1, 3, 7\} \end{aligned}$$

8. (6) Let $x, y \in \mathbb{R}$. Prove, using induction on n that $(xy)^n = x^n y^n$, for all $n \in \mathbb{N}$.

$$\begin{aligned} \text{Base: } (xy)^1 &= xy = x^1 y^1 \\ \text{Ind. hyp: } &\text{Assume } (xy)^k = x^k y^k \text{ for } k \geq 1, k \in \mathbb{N}. \\ \text{Ind. Step: } (xy)^{k+1} &= (xy)^k (xy) \\ &= x^k y^k (xy) \text{ using the ind. hyp.} \\ &= x^k x y^k y \\ &= x^{k+1} y^{k+1} \end{aligned}$$