CISC-102
Fall 2017

Quiz 4 B

November 29, 2017

Student ID: Solutions

Read the questions carefully. Please clearly state any assumptions that you make that are not explicitly stated in the question.

Please answer all questions in the space provided. Use the back of pages for scratch work. NO CALCULATORS. There are 4 pages to this quiz. Note that (x) denotes the question is worth x points.

1. (4) Here are the first few rows of Pascal's triangle:

$$
\begin{array}{ccccccccccccc}
 & & & & & & 1 & & & & & & \\
 & & & & & 1 & & 1 & & & & & \\
 & & & & 1 & & 2 & & 1 & & & & \\
 & & & 1 & & 3 & & 3 & & 1 & & & \\
 & & 1 & & 4 & & 6 & & 4 & & 1 & & \\
 & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & \\
1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1
\end{array}
$$

The next row of Pascal's triangle is: (underline all of the correct answers).

(a) 1 6 20 30 20 6 1

(b) $\underline{\binom{7}{0} \binom{7}{1} \binom{7}{2} \binom{7}{3} \binom{7}{4} \binom{7}{5} \binom{7}{6} \binom{7}{7}}$

(c) $\binom{6}{0} \binom{6}{1} \binom{6}{2} \binom{6}{3} \binom{6}{4} \binom{6}{5} \binom{6}{6}$

(d) 1 6 12 20 12 6 1

(e) <u>1 7 21 35 35 21 7 1</u>

2. Prove using mathematical induction on n that

$$\sum_{m=1}^{n} \binom{m}{m-1} = \binom{n+1}{n-1}$$

is true for all natural numbers n.

(a) (2) The base case for the proof is

Base:

Show, by expanding the binomial coefficients that

$$\sum_{m=1}^{1} \binom{m}{m-1} = \binom{(1+1)}{0}$$

is true.

$$\sum_{m=1}^{1} \binom{m}{m-1} = \binom{1}{0} = \frac{1!}{0!\,1!} = 1$$

$$\binom{1+1}{0} = \binom{2}{0} = \frac{2!}{2!\,0!} = 1$$

(b) (4) Now complete the proof.

Induction hypothesis:

$$\sum_{m=1}^{k} \binom{m}{m-1} = \binom{k+1}{k-1}$$

for $k \geq 1$, $k \in \mathbb{N}$

Induction step:

$$\sum_{m=1}^{k+1} \binom{m}{m-1} = \sum_{m=1}^{k} \binom{m}{m-1} + \binom{k+1}{k}$$

$$= \binom{k+1}{k-1} + \binom{k+1}{k}$$

$$= \binom{k+2}{k}$$

3. (5) Complete the truth table below, adding columns as needed, for the proposition:

$$(p \vee q) \wedge \neg p$$

p	q	$p \vee q$	$(p \vee q) \wedge \neg p$	
T	T	T	F	
T	F	T	F	
F	T	T	T	
F	F	F	F	

4. Consider the logical argument:

$$p \to q, \neg q \vdash \neg p.$$

(a) (2) Rewrite the logical argument as a logical expression.

$((p \to q) \wedge \neg q) \to \neg p$

(b) (4) Complete the truth table below, adding columns as needed to determine whether the argument above is valid or not. After you have completed the table explain your conclusion in a sentence or two.

p	q	$p \to q$	$(p \to q) \wedge \neg q$	$((p \to q) \wedge \neg q) \to \neg p$
T	T	T	F	T
T	F	F	F	T
F	T	T	F	T
F	F	T	T	T

The expression $((p \to q) \wedge \neg q) \to \neg p$ is a tautology. There fore we conclude that the argument $((p \to q) \wedge \neg q) \to \neg p$ is a valid argument.

CISC-102
Fall 2017

Quiz 4 A

November 29, 2017

Student ID: Solutions

Read the questions carefully. Please clearly state any assumptions that you make that are not explicitly stated in the question.

Please answer all questions in the space provided. Use the back of pages for scratch work. NO CALCULATORS. There are 4 pages to this quiz. Note that (x) denotes the question is worth x points.

1. (4) Here are the first few rows of Pascal's triangle:

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}
$$

The next row of Pascal's triangle is: (underline all of the correct answers).

(a) 1 6 20 30 20 6 1

(b) $\binom{7}{0}$ $\binom{7}{1}$ $\binom{7}{2}$ $\binom{7}{3}$ $\binom{7}{4}$ $\binom{7}{5}$ $\binom{7}{6}$ $\binom{7}{7}$

(c) $\binom{6}{0}$ $\binom{6}{1}$ $\binom{6}{2}$ $\binom{6}{3}$ $\binom{6}{4}$ $\binom{6}{5}$ $\binom{6}{6}$

(d) 1 6 12 20 12 6 1

(e) 1 7 21 35 35 21 7 1

2. Prove using mathematical induction on n that

$$\sum_{m=1}^{n} \binom{m}{m-1} = \binom{n+1}{2}$$

is true for all natural numbers n.

(a) (2) The base case for the proof is

Base:

Show, by expanding the binomial coefficients that

$$\sum_{m=1}^{1} \binom{m}{m-1} = \binom{(1+1)}{2}$$

is true.

$$\sum_{m=1}^{1} \binom{m}{m-1} = \binom{1}{0} = \frac{1!}{0!\,1!} = 1$$

$$\binom{(1+1)}{2} = \binom{2}{2} = \frac{2!}{2!\,0!} = 1$$

(b) (4) Now complete the proof.

Induction hypothesis:

$$\sum_{m=1}^{k} \binom{m}{m-1} = \binom{k+1}{2} \quad \text{for } k \geq 1,\ k \in \mathbb{N}$$

Induction Step:

$$\sum_{m=1}^{k+1} \binom{m}{m-1} = \sum_{m=1}^{k} \binom{m}{m-1} + \binom{k+1}{k}$$

$$= \binom{k+1}{2} + \binom{k+1}{k}$$

$$= \binom{k+1}{2} + \binom{k+1}{1}$$

$$= \binom{k+2}{2} \qquad \blacksquare$$

3. (5) Complete the truth table below, adding columns as needed, for the proposition:

$$p \wedge \neg(p \vee q).$$

p	q	$p \vee q$	$\neg(p \vee q)$	$p \wedge \neg(p \vee q)$
T	T	T	F	F
T	F	T	F	F
F	T	T	F	F
F	F	F	T	F

4. Consider the logical argument:

$$p, p \rightarrow q \vdash q.$$

(a) (2) Rewrite the logical argument as a logical expression.

$$(p \wedge (p \rightarrow q)) \rightarrow q$$

(b) (4) Complete the truth table below, adding columns as needed to determine whether the argument above is valid or not. After you have completed the table explain your conclusion in a sentence or two.

p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$p \wedge (p \rightarrow q) \rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

The expression $(p \wedge (p \rightarrow q)) \rightarrow q$ is a ~~tau~~tautology, so we conclude that the argument $p, p \rightarrow q \vdash q$ is valid.