CISC-102 FALL 2017

HOMEWORK 7 SOLUTIONS

Problems

(1) Find all Natural numbers between 1 and 50 that are congruent to 4 (mod 11).

4, 15, 26, 37, 48. You can verify that $11|(4-4), 11|(15-4), 11|(26-4), 11|(37-4)$, and $11|(48-4)$.

(2) Find two Natural numbers a and b such that $2a \equiv 2b(\mod 6)$, but $a \not\equiv b(\mod 6)$.

You can solve this problem using trial and error. A good place to start is $a = 1$ so we have $2(1) \equiv 2(\mod 6)$, The next Natural number that is congruent to $2(\mod 6)$ is 8. So setting $b = 4$ gives us $2(1) \equiv 2(4)(\mod 6)$, and $1 \not\equiv 4(\mod 6)$.

(3) Prove that if $a \equiv b(\mod m)$ and $c \equiv d(\mod m)$ then $a - c \equiv b - d(\mod m)$.

Proof. $a \equiv b(\mod m)$ and $c \equiv d(\mod m)$ respectively imply:

$b - a = pm$ and $d - c = qm$ where p and q are integers.

Therefore we have:

$$\begin{aligned}(b-a)-(d-c) &= pm - qm \\ &= m(p-q)\end{aligned}$$

Therefore we can conclude that $m|(b-a)-(d-c)$ so $a - c \equiv b - d(\mod m)$.

□

(4) Write out each of the 5 residue classes (mod 5) for integers in the range -10 to 10.

$[0]_5 = \{-10, -5, 0, 5, 10\}$

$[1]_5 = \{-9, -4, 1, 6\}$

$[2]_5 = \{-8, -3, 2, 7\}$

$[3]_5 = \{-7, -2, 3, 8\}$

$[4]_5 = \{-6, -1, 4, 9\}$