CISC-102 FALL 2017

HOMEWORK 8
SOLUTIONS

(1) What is the number of ways to colour n identical objects with 3 colours? What is the number of ways to colour n identical objects with 3 colours so that each colour is used at least once?

The number of ways to colour n objects with 3 colours can be viewed as counting the number of binary strings with n 0's and 2 1's. This yields the expression:

$$\frac{(n+2)!}{n!2!} = \binom{n+2}{2} = \binom{n+2}{n}$$

To ensure that each colour is used at least once we pre-assign one object per colour leaving $n-3$ objects to be coloured with no further restrictions. We map this problem to counting binary strings with $n-3$ 0's and 2 1's. This yields the expression:

$$\frac{(n-3+2)!}{(n-3)!2!} = \binom{n-1}{2} = \binom{n-1}{n-3}$$

(2) How many different strings can you make using the letters TIMBITS?

The are $\frac{7!}{2!^2}$ different strings.

(3) How many 5 card hands are there (unordered selection from a standard 52 card deck) that consist of a single pair of the same value, and three other cards of different values? Two possible examples are:

$$2\heartsuit, 2\diamondsuit, 7\clubsuit, 9\diamondsuit 3\heartsuit \text{ and } A\heartsuit, A\clubsuit, 4\diamondsuit, 6\diamondsuit 3\heartsuit$$

First consider the pair. There are 13, or $\binom{13}{1}$ possible values for the pair. Within each value there are $\binom{4}{2}$ ways to select the suits of the cards.

The remaining three cards must come from the remaining 12 choices. There are $\binom{12}{3}$ ways of getting those 3 values without regard to the suit. There are 4, or $\binom{4}{1}$ ways to select the suit for each of these three cards.

Putting this all together we get the product:

$$\binom{13}{1}\binom{4}{2}\binom{12}{3}\binom{4}{1}^3.$$

(4) From 100 used cars siting on a lot, 20 are to be selected for a test designed to check safety requirements. These 20 cars will be returned to the lot, and again 20 will be selected for testing for emission standards.

(a) In how many ways can the cars be selected for safety requirement testing?

$$\binom{100}{20}$$

(b) In how many ways can the cars be selected for emission standards testing?

$$\binom{100}{20}$$

(c) In how many different ways can the cars be selected for both tests?

$$\binom{100}{20}\binom{100}{20}$$

(d) In how many ways can the cars be selected for both tests if exactly 5 cars must be tested for safety and emission?

$$\binom{100}{5}\binom{95}{15}\binom{80}{15}$$