

## CISC-102 WINTER 2019

### HOMEWORK 2

Please work on these problems and have your solutions ready by next Thursday. Assignments will **not** be collected for grading.

### READINGS

Read sections 1.5, 1.6, 1.7, 1.8 of *Schaum's Outline of Discrete Mathematics*.

Read sections 1.1, 1.2 and 1.3 again (if you did not understand things last week) of *Discrete Mathematics Elementary and Beyond*.

### PROBLEMS

- (1) Illustrate DeMorgan's Law  $(A \cap B)^c = A^c \cup B^c$  using Venn diagrams.
- (2) Let  $A_i = \{1, 2, 3, \dots, i\}$  for all  $i \in \mathbb{N}$ . For example  $A_4 = \{1, 2, 3, 4\}$ .

(a) What are the elements of the set

$$\cup_{i=1}^n A_i?$$

(b) What are the elements of the set

$$\cap_{i=1}^n A_i?$$

- (3) Observe that  $A \subseteq B$  has the same meaning as  $A \cap B = A$ . Draw a Venn diagram to illustrate this fact.
- (4) Use a Venn diagram to show that if  $A \subseteq B$  **and**  $B \subseteq C$ , then  $A \subseteq C$ .
- (5) Use the Principle of Exclusion and Inclusion to show that  $|A \cup B| + |A \cap B| = |A| + |B|$ . (It may help your understanding if you first explore an example such as  $A = \{1, 2, 3\}$  and  $B = \{3, 4\}$ )
- (6) What are the cardinalities of the following sets?
  - (a)  $A = \{\text{winter, spring, summer, fall}\}$ .
  - (b)  $B = \{x : x \in \mathbb{Z}, 0 < x < 7\}$ .
  - (c)  $\mathcal{P}(B)$ , that is, the power set of  $B$ .
  - (d)  $C = \{x : x \in \mathbb{N}, x \text{ is even}\}$
- (7) Suppose that we have a sample of 100 students at Queen's who take at least one of the following language courses, French-101, Spanish-101, German-101. Also suppose that 65 take French-101, 45 take German-101, 42 take Spanish 101, 20 take French-101 and German-101, 25 take French-101 and Spanish-101, and 15 take German-101 and Spanish-101.

- (a) How many students take all three language courses? (HINT: Use the Principle of Inclusion and Exclusion to write an expression representing these students and the classes they take. )
  - (b) Draw a Venn diagram representing these 100 students and fill in the regions with the correct number.
  - (c) How many students take exactly 1 of these courses?
  - (d) How many students take exactly 2 of these courses?
- (8) At an art class with 30 students, there are 14 women, and 16 men. Twenty-two of the students are right-handed. What is the minimum and maximum number of women that are right-handed?
- (9) Recall that the union operation is associative, that is  $A \cup (B \cup C) = (A \cup B) \cup C$ . Show that the relative complement set operation is not associative, that is,  $A \setminus (B \setminus C) = (A \setminus B) \setminus C$ , is incorrect for some sets A, B, C. (Note if relative complement is associative then the equation must be true for all sets A, B, C.)
- (10) Consider a set  $S$  of  $n$  elements, such that  $\{a, b\} \subseteq S$ .
- (a) What is the cardinality of the power set of  $S \setminus \{a\}$ ?
  - (b) What is the cardinality of the power set of  $S \setminus \{a, b\}$ ?
  - (c) How many subsets of  $S$  are there that contain the element  $a$ ?
  - (d) How many subsets of  $S$  are there that contain the element  $a$  and exclude the element  $b$ ?