CISC-102 FALL 2019

HOMEWORK 5 SOLUTIONS

(1) Consider the following relations on the set $A = \{1, 2, 3\}$:

- $R = \{(1, 1), (1, 2), (1, 3), (3, 3)\}$,
- $S = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$,
- $T = \{(1, 1), (1, 2), (2, 2), (2, 3)\}$,
- $A \times A$.

For each of these relations determine whether it is symmetric, antisymmetric, reflexive, or transitive.

S and A $\times$ A are symmetric.

R and T are antisymmetric.

S and A $\times$ A are reflexive.

R, S and A $\times$ A are transitive.

(2) Explain why each of the following binary relations on the set $S = \{1, 2, 3\}$ is or is not an equivalence relation on S.

(a) $R_1 = \{(1, 1), (1, 2), (3, 2), (3, 3), (2, 3), (2, 1)\}$

(b) $R_2 = \{(1, 1), (2, 2), (3, 3), (2, 1), (1, 2),$
$(3, 2), (2, 3), (3, 1), (1, 3)\}$

(c) $R_3 = \{(1, 1), (2, 2), (3, 3), (3, 1), (1, 3)\}$

R_1, is neither reflexive nor transitive so it's not an equivalence relation. R_1 is symmetric.

R_2 is reflexive, symmetric, and transitive so it is an equivalence relation.

R_3 is reflexive, symmetric and transitive, so it is an equivalence relation.

(3) Let R be a relation on the set of Natural numbers such that $(a, b) \in$ R if $a \geq b$. Show that the relation R on $\mathbb{N}$ is a partial order.

R is reflexive because for all $a \in (N)$ $a \geq a$. R is antisymmetric because for all $a, b \in \mathbb{N}, a \neq b$ we have either $a \geq b$ or $b \geq a$ but not both. R is transitive because for all $a, b, c \in \mathbb{N}$, if $a \geq b$ and $b \geq c$, we have $a \geq c$.

(4) Which of the following relations on the set $S = \{1, 2, 3, 4, 5, 6\}$ is a function?

- R = {(1,1), (2,2), (3,2), (4,2), (5,3),(6,3) }
- S = {(1,1), (2,2), (3,2), (4,2), (5,3),(6,3), (1,4) }
- T = {(1,1), (2,2), (3,3), (4,4) }
- $S \times S$

R is a function, because every element in the domain, S, has a distinct image.

S is not a function, because 1 has two different images, due to the pairs (1,1), and (1,4).

T is not a function because the elements of S 5 and 6 do not have images.

$S \times S$ is not a function because every element of S has multiple images.