

## CISC-102 FALL 2019

### HOMEWORK 7 SOLUTIONS

- (1) Find all Natural numbers between 1 and 50 that are congruent to 4 (mod 11).

Observe if  $x$  is a natural number such that  $x \equiv 4 \pmod{11}$  then  $11|(x-4)$ , or equivalently the difference  $x-4$  is a multiple of 11. So our list is 4, 15, 26, 37, 48.

A succinct way to describe all natural numbers  $x$  such that  $x \equiv 4 \pmod{11}$  is  $[4]_{11}$ .

- (2) Find two Natural numbers  $a$  and  $b$  such that  $2a \equiv 2b \pmod{6}$ , but  $a \not\equiv b \pmod{6}$ .

A systematic and easy way to solve this is to consider equivalence classes mod 6. In particular  $[0]_6 = 6, 12, \dots$ . We have  $6 = 2 \times 3$ , and  $12 = 2 \times 6$ , which immediately gives us our solution  $a = 3$  and  $b = 6$ .

- (3) Prove that if  $a \equiv b \pmod{m}$  and  $c \equiv d \pmod{m}$  then  $a - c \equiv b - d \pmod{m}$ .

Observe that we need to show that  $m|((a-c) - (b-d))$ . Thus,  $a \equiv b \pmod{m}$  and  $c \equiv d \pmod{m}$  respectively imply that  $m|(a-b)$  and  $m|(c-d)$ , and  $m|((a-b) - (c-d))$ , and this can be rewritten as:  $m|(a-c) - (b-d)$ .

- (4) Write out each of the 5 residue classes (mod 5) for integers in the range -10 to 10.

$$[0]_5 = \{-10, -5, 0, 5, 10\}$$

$$[1]_5 = \{-9, -4, 1, 6\}$$

$$[2]_5 = \{-8, -3, 2, 7\}$$

$$[3]_5 = \{-7, -2, 3, 8\}$$

$$[4]_5 = \{-6, -1, 4, 9\}$$

- (5) Let  $S$  be a finite subset of the positive integers. What is the smallest value for  $|S|$  that guarantees that at least two elements  $x, y \in S$  have the same remainder when divided by 100. HINT: Use the pigeon hole principle.

There are 100 residue classes (mod 100) so by the pigeon hole principle any subset of the positive integers that has at least 101 elements has two or more elements that are in the same residue class, or equivalently have the same remainder when divided by 100.

- (6) Prove that any set of 5 natural numbers will always have two numbers  $n_1$  and  $n_2$  such that  $4|(n_1 - n_2)$ . Hint: Use the Pigeon Hole Principle.

There are exactly 4 residue classes mod 4. By the pigeon whole principle any 5 natural numbers will have  $n_1$  and  $n_2$  (and possibly  $n_1 = n_2$ ) such that  $n_1$  and  $n_2$  are in the same residue class.

- (7) New parents wish to give their new baby one, two, or three different names. They have a book containing 500 names that they will choose from. How many different ways can this baby be named?

If the baby is given one name then there are obviously 500 choices.  
 If the baby is given two names then we use the product rule to deduce that there are  $500 \times 499$  choices.  
 If the baby is given three names then we use the product rule to deduce that there are  $500 \times 499 \times 498$  choices.

We now use the sum rule to add up the choices to get a total of  $500 + 500 \times 499 + 500 \times 499 \times 498$  choices.

- (8) You have chosen a password that consists of 4 upper case letters from a 26 letter alphabet. How many passwords does a hacker have to try to be sure that they can break in? What if you may use both upper and lower case for your four symbol password? (Note: You may use upper and lower case letters, but that does not preclude the possibility that all of the letters are upper case or lower case.) Finally consider a password 7 symbols long and you may use both upper and lower case letters, and at least one digit (0...9).

There are  $26^4$  different 4 symbol passwords using only upper case letters.  
 There are  $52^4$  different 4 symbol passwords using both upper and lower case letters.  
 There are  $62^7$  different 7 symbol passwords upper and lower case letters and digits.  
 But this also counts passwords with no digits. Therefore we subtract the number of passwords using upper and lower case letters, but no digits, that is,  $62^7 - 52^7$ .

- (9) How many different strings can you make using the letters TIMBITS?

The are  $\frac{7!}{2!^2}$  different strings.