CISC-102 WINTER 2019

HOMEWORK 10

Assignments will **not** be collected for grading.

Readings

Read chapter 4 of *Schaum's Outline of Discrete Mathematics*.
Read section 3.1, 3.5 and 3.6 of *Discrete Mathematics Elementary and Beyond*.

Problems

(1) Prove (using mathematical induction on n) that:

$$\sum_{m=0}^{n-1} \binom{m+1}{m} = \binom{n+1}{n-1}$$

is true for all $n \in \mathbb{N}$.

(2) Use a truth table to verify that the proposition $p \vee \neg(p \wedge q)$ is a tautology, that is, the expression is true for all values of p and q.

(3) Use a truth table to verify that the proposition $(p \wedge q) \wedge \neg(p \vee q)$ is a contradiction, that is, the expression is false for all values of p and q.

(4) Use a truth table to show that $p \vee q \equiv \neg(\neg p \wedge \neg q)$

(5) Show that the following argument is valid.

$$p \to q, \neg q \vdash \neg p$$

(6) Let A = {1,2,3,4,5}. Determine the truth value of each of the following statements.
   (a) $(\exists x \in A)(x + 2 = 7)$
   (b) $(\forall x \in A)(x + 2 < 8)$
   (c) $(\exists x \in A)(x + 3 < 2)$
   (d) $(\forall x \in A)(x + 3 \leq 9)$

(7) Let $A = \{1, 2, 3, 4, 5\}$. And let $(x, y) \in A^2$, be the domain of the propositions given below. Determine the truth value of the following statements.
   (a) $\exists x \forall y, x^2 < y + 1$
   (b) $\forall x \exists y, x^2 < y + 1$