CISC-102 WINTER 2019

HOMEWORK 3

Readings

Read sections 1.8 of *Schaum's Outline of Discrete Mathematics*.
Read section 2.1 of *Discrete Mathematics Elementary and Beyond*.

Problems

(1) Prove using mathematical induction that the sum of the first n natural numbers is equal to $\frac{n(n+1)}{2}$. This can also be stated as:
Prove that the proposition P(n),

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

is true for all $n \in \mathbb{N}$

(2) Prove using mathematical induction that the proposition P(n),

$$\sum_{i=2}^{n} i = \frac{(n-1)(n+2)}{2}$$

is true for all $n \in \mathbb{N}, n \geq 2$.

(3) Prove using mathematical induction that the proposition P(n),

$$\sum_{i=3}^{n} i = \frac{(n-2)(n+3)}{2}$$

is true for all $n \in \mathbb{N}, n \geq 3$.

(4) Prove using mathematical induction that the proposition P(n)

$$n! \leq n^n$$

is true for all $n \in \mathbb{N}$.

(5) Let P(n) be the proposition that if $A_1, A_2, \ldots A_n, B_1, B_2, \ldots, B_n$ are sets such that $A_i \subseteq B_i$ for all $i, 1 \leq i \leq n$, then $\cap_{i=1}^{n} A_i \subseteq \cap_{i=1}^{n} B_i$. Prove, using mathematical induction that P(n) is true for all natural numbers $n \geq 2$.

(6) Given a set of n points on a two dimensional plane, such that no three points are on the same line, it is always possible to connect every pair of points with a line segment. The figure illustrates this showing 5 points, that are pairwise connected with 10 line segments. Prove using mathematical induction that the total number of line segments is $\frac{n(n-1)}{2}$ for any number of points $n \in \mathbb{N}, n \geq 2$.

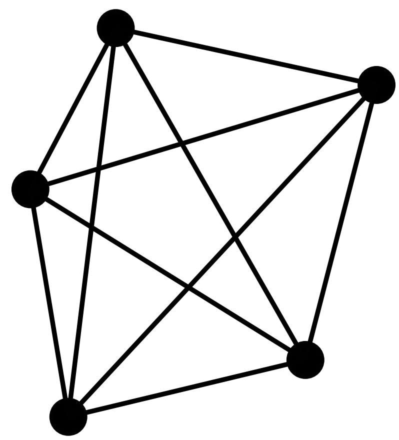

Figure 1. Five points, pairwise connected with 10 line segments.