CISC-102 WINTER 2019

HOMEWORK 6

Assignments will **not** be collected for grading.

Readings

Read sections 11.1, 11.2, 11.3, 11.4, and 11.5 of *Schaum's Outline of Discrete Mathematics.*

Read section 6.1, and 6.2 of *Discrete Mathematics Elementary and Beyond.*

Problems

(1) Find the quotient q and remainder r, as given by the Division Algorithm theorem for the following examples.
  (a) a = 500, b = 17
  (b) a = -500, b = 17
  (c) a = 500, b = -17
  (d) a = -500, b = -17

(2) Show that $c|0$, for all $c \in \mathbb{Z}, c \neq 0$.

(3) Show that $1|z$ for all $z \in \mathbb{Z}$.

(4) Use the fact that if $a|b$ and $b \neq 0$ then $|a| \leq |b|$ to prove that if $a|b$ and $b|a$ then $|a| = |b|$.

(5) Use the previous two results to prove that if $a|1$ then $|a| = 1$.

(6) Let $a, b, c \in \mathbb{Z}$ such that $c|a$ and $c|b$. Let r be the remainder of the division of b by a, that is there is a $q \in \mathbb{Z}$ such that $b = qa + r, 0 \leq r < |b|$. Show that under these condition we have $c|r$.

(7) Consider the function A, such that A(1) = 1, A(2) = 2, A(3) = 3, and for $n \in \mathbb{N}, n \geq 4$, $A(n) = A(n-1) + A(n-2) + A(n-3)$.
  (a) Find values $A(n)$ for $n = 4, 5, 6$.
  (b) Use the second form of mathematical induction to prove that $A(n) \leq 3^n$ for all natural numbers n.

(8) Let a = 1763, and b = 42
  (a) Find $\gcd(a, b)$. Show the steps used by Euclid's algorithm to find $\gcd(a, b)$.
  (b) Find integers x, y such that $\gcd(a, b) = ax + by$
  (c) Find lcm(a,b)

(9) Prove $\gcd(a, a+k)$ divides k.

(10) If a and b are relatively prime, that is $\gcd(a, b) = 1$ then we can always find integers x, y such that $1 = ax+by$. This fact will be useful to prove the following proposition.

Suppose p is a prime such that $p|ab$, that is p divides the product ab, then $p|a$ or $p|b$.