CISC-102 WINTER 2019

HOMEWORK 9

Readings

Read sections 5.3 of *Schaum's Outline of Discrete Mathematics.*
Read sections 3.5, 3.6 and 4.1 of *Discrete Mathematics Elementary and Beyond.*

Problems

(1) Consider the equation

$$\sum_{i=0}^{2} \binom{3}{i}\binom{2}{2-i} = \binom{5}{2}. \tag{1}$$

(a) Use algebraic manipulation to prove that the left hand and right hand sides of equation (1) are in fact equal.

(b) Use a counting argument to prove that the left hand and right hand sides of equation (1) are in fact equal.

(2) Now consider a generalization of the previous equation.

$$\sum_{i=0}^{k} \binom{m}{i}\binom{n}{k-i} = \binom{m+n}{k}. \tag{2}$$

Use a counting argument to prove that the left hand and right hand sides of equation (2) are in fact equal.

(3) In the notes for Week 9 you will find Pascal's triangle worked out for rows 0 to 8. The numbers in row 8 are 1 8 28 56 70 56 28 8 1. Work out the values of rows 9 and 10 of Pascal's triangle with the help of the equation:

$$\binom{n-1}{k} + \binom{n-1}{k-1} = \binom{n}{k}. \tag{3}$$

(4) Show that $\binom{n}{0} = \binom{n-1}{0}$, and that $\binom{n-1}{n-1} = \binom{n}{n}$ by an algebraic argument as well as a counting argument.

(5) Prove that

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \cdots + (-1)^n \binom{n}{n} = 0$$

Note that this equation can also be written as follows:

$$\sum_{i=0}^{n} \binom{n}{i} (-1^i) = 0$$

HINT: This can be viewed as a special case of the binomial theorem.

(6) Prove that:

$$\frac{n^k}{k^k} \leq \binom{n}{k} \leq \frac{n^k}{k!}$$

Note: All you need is a bit of algebra for this one.

(7) Let $F(n)$ denote the n^{th} Fibonacci number. Prove, using mathematical induction, that

$$\sum_{i=1}^{n} F(2i-1) = F(2n)$$

For all natural numbers n.

(8) Prove, using mathematical induction, that

$$\sum_{i=1}^{n} F(i)^2 = F(n)F(n+1)$$

For all natural numbers n.