1. (2) Let S={a,b,c,d,e,f,g}. Which one of the following is not a partition of S:

   (a) P1 = [{a,c,e},{f,b},{d,g}]
   (b) P2 = [{a,b,e,g},{c},{d,f}]
   (c) P3 =[{a,e,g},{c,d},{b,e,f}] $\{a,e,g\} \cap \{b,e,f\} \neq \emptyset$
   (d) P4= [{a,b,c,d,e,f,g}]

2. (2) Let A and B be sets such that $A = \{1, 2, 3, 5, 9\}$ and $B = \{2, 4, 6, 8\}$. Which one of the following statements is true?

   (a) $A \subseteq B$
   (b) $A \cap B = 2$
   (c) $A = B^c$
   (d) $|B| \leq |A|$

3. (2) If set $A = \{a, b, c\}$, then what is $|P(A)|$, the cardinality of the powerset of A, or in plain English the number of subsets which can be formed from A?

   (a) 8 $= 2^3$
   (b) 3
   (c) 9
   (d) not enough information

4. (2)Consider the function $f : \mathbb{Z} \to \mathbb{Z}$ such that $f(x) = 6$. Which one of the following statements is true?

   (a) f is a one-to-one function.
   (b) f is an onto function.
   (c) f is neither one-to-one or onto.
   (d) f is a bijection, that is f is both one-to-one and onto.

5. (2) Consider the recursive function defined as $F(n) = F(n - 1) + 7$. What is the value of $F(4)$?

   (a) 28
   (b) 29
   (c) F(3) + 7
   (d) not enough information No base

6. Determine which of the following is true or false.

   (a) (1) $\{3\} \subseteq \{1, 3, 5\}$ T

   (b) (1) $\{3\} \subset \{1, 3, 5\}$ T

   (c) (1) $\{a, b\} \cup \{b, c\} = \{a, b, c\}$ T

   (d) (1) $\{a, b\} \cap \{b, c\} = \{a, b, c\}$ F

   (e) (1) $\emptyset \in \{x, y, z\}$ F

   (f) (1) $\emptyset \subseteq \{x, y, z\}$ T

   (g) (1) if $A = A \cup B$ then $A \subseteq B$ F

   (h) (1) if $A = A \cap B$ then $A \subseteq B$ T

7. Recall that the *relative complement* of a set B with respect to set A is denoted by $A \setminus B$ and the *symmetric difference* of sets A and B is denoted by $A \oplus B$ are define as:

$$A \setminus B = \{x : x \in A, x \notin B\}, A \oplus B = (A \cup B) \setminus (A \cap B).$$

   Letting $\mathbb{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 4\}$, $B = \{1, 2, 3, 5\}$, and $C = \{3, 4\}$.

   (a) (3)What is $A \setminus B$?

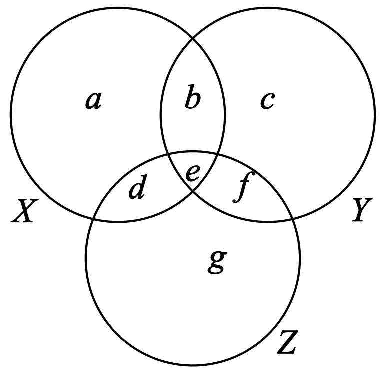
   $A \setminus B = \{4\}$

   (b) (3)What is $A \oplus B$?

   $A \oplus B = \{4, 5\}$

   (c) (3) What is $(B \setminus C) \oplus A$?

   $B \setminus C = \{1, 2, 5\} \oplus A = \{3, 4, 5\}$



8. Consider sets $X, Y Z$ such that $|X| = |Y| = |Z| = 15$, $|X \cap Y| = |X \cap Z| = |Y \cap Z| = 6$ and $|X \cup Y \cup Z| = 28$.

   (a) (4) What is the value of $|X \cap Y \cap Z|$?

   By the Principle of Inclusion & Exclusion:

   $$28 = 3(15) - 3(6) + |X \cap Y \cap Z|$$

   so $|X \cap Y \cap Z| = 1$

   (b) (4) Now write the appropriate value for each of the regions of the Venn diagram drawn above.

   a 4
   b 5
   c 4
   d 5
   e 1
   f 5
   g 4

9. Consider the proposition $P(n)$

$$2^0 + 2^1 + \cdots + 2^{n-2} + 2^{n-1} = 2^n - 1.$$

Answer the following questions to prove that $P(n)$ is true for all $n \in \mathbb{N}$.

(a) (2) What is the base case?

Base: $n=1$ $\quad 2^0 = 2^1 - 1 = 1$

(b) (1) What is the induction hypothesis?

Ind Hyp: $\quad 2^0 + 2^1 + \cdots 2^{k-1} = 2^k - 1$

(c) (3) What is the induction step?

Ind Step:

$$2^0 + 2^1 + \cdots + 2^{k-1} + 2^k = 2^k + 2^k - 1 \quad \text{(Ind Hyp.)}$$
$$= 2^{k+1} - 1$$

$\boxtimes$

10. Consider a recursive function defined as:
$T(1) = 0$
$T(n) = T(n-1) + (n-1)$ for $n \in \mathbb{N}, n \geq 2$.

(a) (3) What are the values for $T(2), T(3)$, and $T(4)$?

$T(2) = 0 + 1 = 1, \; T(3) = 1 + 2 = 3$
$T(4) = 3 + 3 = 6$

(b) (6) Let $P(n)$ be the proposition $T(n) = \frac{n(n-1)}{2}$. Use mathematical induction to prove that $P(n)$ is true for all $n \in \mathbb{N}$.

Base: $T(1) = \frac{1(0)}{2} = 0$

Ind. Hyp: $T(k) = \frac{k(k-1)}{2}$ for $k \in \mathbb{N}$, $k \geq 1$

Ind. Step:

$$T(k+1) = T(k) + k$$
$$= \frac{k(k-1)}{2} + k \quad (\text{Ind. Hyp.})$$
$$= \frac{k^2 - k + 2k}{2}$$
$$= \frac{k^2 + k}{2} = \frac{(k+1)(k)}{2} \quad \blacksquare$$