

CISC-102 WINTER 2019

HOMEWORK 1 SOLUTIONS

PROBLEMS

- (1) Rewrite the following statements using set notation, and then give an example by listing members of sets that match the description. For example: A is a subset of C. Answer: $A \subseteq C$. $A = \{1, 2\}$, $C = \{1, 2, 3\}$.

There are many different solutions to these questions. I have shown several possibilities.

- (a) The element 1 is not a member of (the set) A.
 $1 \notin A$. $A = \{2, 4\}$.
- (b) The element 5 is a member of B.
 $5 \in B$. $B = \{5, 6\}$
- (c) A is not a subset of D.
 $A \not\subseteq D$. $A = \{2, 4\}$ and $D = \{42, 18\}$.
- (d) E and F contain the same elements.
 $E = F$. $E = F = \{7\}$. $E \subseteq F$ and $F \subseteq E$.
- (e) A is the set of integers larger than three and less than 12.
 $A = \{x : x \in \mathbb{Z}, 3 < x < 12\}$. $A = \{4, 5, 6, 7, 8, 9, 10, 11\}$.
- (f) B is the set of even natural numbers less than 15.
 $B = \{2x : x \in \mathbb{N}, x < 8\}$. $B = \{2, 4, 6, 8, 10, 12, 14\}$.
- (g) C is the set of natural numbers x such that $4 + x = 3$.
 $C = \{x : x \in \mathbb{N}, 4 + x = 3\}$. $C = \emptyset$.
- (2) $A = \{x : 3x = 6\}$. $A = 2$, true or false? $A = \{2\}$. $A \neq 2$, so the statement is false.
- (3) Which of the following sets are equal $\{r, s, t\}$, $\{t, s, r\}$, $\{s, r, t\}$, $\{t, r, s\}$. They are all equal. The order in which elements are written in a set is not important, unless ellipses "... " are used to denote a sequence. For example $x = \{1, 2, \dots, 10\}$.

- (4) Consider the sets $\{4, 2\}$, $\{x : x^2 - 6x + 8 = 0\}$, $\{x : x \in \mathbb{N}, x \text{ is even}, 1 < x < 5\}$. Which one of these sets is equal to $\{4, 2\}$?

They are all equal.

- (5) Which of the following sets are equal: \emptyset , $\{\emptyset\}$, $\{0\}$. None are equal. $\{\emptyset\}$ is a set within a set. 0 is a number not a set, and definitely not the empty set.
- (6) Explain the difference between $A \subseteq B$, and $A \subset B$, and give example sets that satisfy the two statements.

$A \subseteq B$ is pronounced as “A is a subset of B” implying that A is a subset of B that may also be equal to A. $A = B = \{1\}$. $A \subset B$ is pronounced “A is a proper subset of B ” implying that A is strictly a subset of B. $A = \{1\}$, $B = \{1, 2\}$.

- (7) Consider the following sets $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 4, 5, 6, 7\}$, $C = \{3, 4\}$, $D = \{4, 5, 6\}$, $E = \{3\}$.

- (a) Let X be a set such that $X \subseteq A$ and $X \subseteq B$. Which of the sets could be X ? For example X could be C , or X could be E . Are there any other sets that could be X ?

X could also be $\{2, 3, 4\}$.

- (b) Let $X \not\subseteq D$ and $X \not\subseteq B$. Which of the the sets could be X ? Set A is the only set from the list that is not a subset of D and not a subset of B . There are infinitely more possibilities of sets that satisfy these requirements. For example all sets $X_i = \{x : x \in \mathbb{N}, x > 8 + i\}$ for all values of $i \in \mathbb{N}$, represents an infinite collection of sets that are not subsets of B or D .

- (c) Find the smallest set M that contains all five sets.

$$M = \{1, 2, 3, 4, 5, 6, 7\}$$

- (d) Find the largest set N that is a subset of all five sets. $N = \emptyset$

- (8) Is an “element of a set”, a special case of a “subset of a set”?

No, an element of a set is not a subset.

- (9) Phrase the handshake counting problem using set theory notation.

How many two element subsets can be chosen from an n element set?

(10) List all of the subsets of $\{1, 2, 3\}$.

$\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$.

(11) Let $A = \{a, b, c, d, e\}$. List all the subsets of A containing a but not containing b .

$\{a\}, \{a,c\}, \{a,d\}, \{a,e\}, \{a,c,d\}, \{a,c,e\}, \{a,d,e\}, \{a,c,d,e\}$