CISC-868 FALL 2011

HOMEWORK 2

These questions come from *Discrete and Computational Geometry* by Satyan Devadoss and Joseph O' Rourke.

Exercise 2.1: Show that the use of the word convex in convex hull is justified; that is, show that conv(S) is indeed a convex region.

Exercise 2.3: Let S be the four points (0, 0), (0, 1), (1, 0), (1, 1) in the plane. Show using Theorem 2.2 (*For a point set S the convex hull of S is the set of all convex combinations of S.*) that conv(S) is the square with vertices at S.

Exercise 2.5: Show that conv(S) is the convex polygon with the smallest perimeter that contains S.

Exercise 2.6: Show that conv(S) is the convex polygon with the smallest area containing S.

Exercise 2.10: Prove that if S is the set of n points sampled from a uniform distribution in a unit square, then the expected number of points on the hull of S is of order $O(\log n)$. $\star$ Note: that the $\star$ denotes that this problem is particularly challenging.

Exercise 2.18: We phrased the gift-wrapping algorithm in terms of angle comparisons, which are notoriously slow and numerically unstable when implemented naively. Show that angle comparisons can be replaced by LEFT-OF tests, where LEFT-OF(a, b, c) is true exactly when c is left of the directed line through a and b. Write the gift wrapping algorithm described in the text in section 2.4 using the style of my notes from week 1. Also note that the function *orientation* that I used for the convex hull algorithms is equivalent to the LEFT-OF tests.

Exercise 2.19: Describe a point set with n points that serves as the worst-case for the gift-wrapping algorithm.

Exercise 2.20: Describe a point set with n points that constitutes the best-case for the gift-wrapping algorithm. What is its time complexity in this case?