Name: _____________Solutions__________________ Student Number: _____________________________	CISC 203 Discrete Mathematics for Computing Science Test 5, Fall 2009 Professor Mary McCollam

Please write in pen and only in the box marked "Answer".
This is a closed-book exam. No computers or calculators are allowed.

Question 1: [10 marks]

(a) [6 marks] Show that the relation
$D = \{ (x,y) \mid x - y \text{ is an integer} \}$
Is an equivalence relation on the set of real numbers.

Answer:

The relation is reflexive, since $x - x$ is an integer; thus, $(x, x) \in D$ for all x.

The relation is symmetric, since if $(x, y) \in D$, then $x - y$ is an integer, say z. Therefore, $y - x$ is an integer $- z$. Thus $(y, x) \in D$.

The relation is transitive for the following reason. Suppose $(w, x) \in D$ and $(x, y) \in D$. Then $w - x = z_1$ and $x - y = z_2$ for integers z_1 and z_2.
Therefore, $(w - x) + (x - y) = (w - z) = z_1 + z_2$.
Since $z_1 + z_2$ is an integer, then $(w, y) \in D$.

(b) [4 marks] Describe the equivalence class of each of 0 and 0.5 for the relation *D*.

Answer:

The equivalence class for 0 is the set of integers Z (since k is related to 0 if and only if $k - 0$ is an integer).

The equivalence class for .5 is the set k +.5 for all integers k (since any arbitrary number x is related to 0.5 if and only if $x - 0.5$ is an integer, and hence x must be of the form $k + 0.5$ for all integers k.

Question 2: [10 marks]

a) [4 marks] Is ($\mathcal{Z}$, ≥) a poset, where $\mathcal{Z}$ is the set of integers? Why or why not?

Answer:

We must show that the relation ≥ on the set of integers is reflexive, antisymmetric, and transitive.

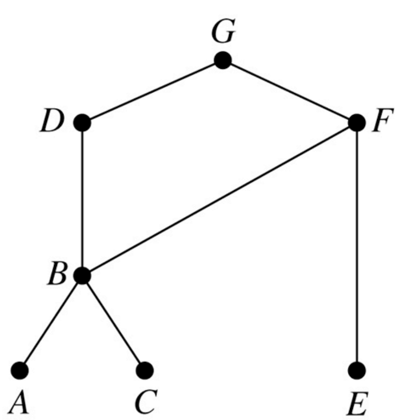
Because a ≥ a for every integer a, ≥ is reflexive.

If a ≥ b and b ≥ a, then a = b. Hence ≥ is antisymmetric.

Finally, ≥ is transitive because a ≥ b and b ≥ c imply that a ≥ c.

It follows that ≥ is a partial ordering on the set of integers and ($\mathcal{Z}$, ≥) is a poset.

b) [6 marks] In the poset represented by the Hasse diagram below, identify the:



Answer:

i) maximal and minimal elements

maximal: G minimal: A, C, and E

ii) greatest and least elements, if they exist

greatest: G There is no least element.

iii) upper bounds of { b, c }

B, D, F, and G

iv) least upper bound of { b, c } if it exists

B

v) lower bounds of { d, g, f }

A, B, C

vi) greatest lower bound of { d, g, f } if it exists

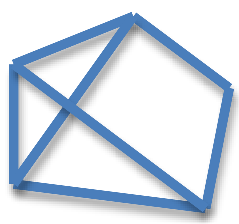
B

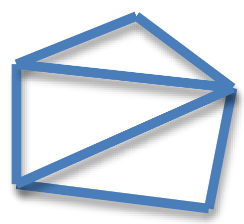
Question 3: [10 marks]

a) Draw the two undirected graphs represented by the following adjacency matrices.

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Answer:





b) Determine whether these two graphs are isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.

Answer:

The two graphs are not isomorphic.

There is one vertex of degree 4 in the second graph, but no vertex of degree 4 in the first graph.

Question 4: [10 marks]

a) Which complete bipartite graphs $K_{m,n}$, where m and n are positive integers, are trees?

Answer:

$K_{1,n}$ is a tree.

Also, $K_{m,1}$ is a tree. No other complete bipartite graphs are trees.

So, $K_{m,n}$ is a tree if and only if $m = 1$ or $n = 1$.

b) Show the result of inserting, 8, 5, 7, 3, 4, 9, 2 sequentially (one at a time), in an initially empty binary search tree.

Answer:

```
          8
        /   \
      5       9
    /   \
  3       7
 / \
2   4
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