	CISC 203
	Discrete Mathematics for Computing Science
Name:_____	
	Test 1
Student Number:_____	Fall 2010
	Professor Mary McCollam

This test is 50 minutes long and there are 40 marks.

Please write in pen and only in the box marked "Answer".

This is a closed-book exam. No computers or calculators are allowed.

Write any assumptions you are making when answering a question.

Question 1:	/ 10
Question 2:	/ 10
Question 3:	/ 10
Question 4:	/ 10
Total:	/ 40

Question 1: [10 marks]

(a) Let $\mathcal{A} = \{x \mid -10 \le x \le 10\}$, $\mathcal{B} = \{x \mid -15 \le x \le 8\}$ and $C = \{x \mid 2 \le x \le 15\}$. Let the universe of discourse be $\mathcal{U} = Z$, the set of integers. Determine the following set.

Answer:

$$(\mathcal{B} - C) \cap \mathcal{A} =$$

(b) Let $\mathcal{A}_i = \{ \ldots, -2, -1, 0, 1, 2, \ldots, i \}$. Determine each of the following sets.

Answer:

i)

$$\bigcup_{i=1}^{n} \mathcal{A}_i = \mathcal{A}_1 \cup \mathcal{A}_2 \cup \mathcal{A}_3 \cup \ldots \cup \mathcal{A}_n =$$

ii)

$$\bigcap_{i=1}^{n} \mathcal{A}_i = \mathcal{A}_1 \cap \mathcal{A}_2 \cap \mathcal{A}_3 \cap \ldots \cap \mathcal{A}_n =$$

Question 2: [10 marks]

(a) [4 marks] Determine whether the function $f(x) = 2x - 1$ is a bijection (one-to-one correspondence) from the set of positive integers to the set of positive integers Justify your answer.

Answer:

(b) [3 marks] What is the inverse of $f(x) = 6 - 3x^{1/2}$? You do not have to show that your result is correct.

Answer:

(c) [3 marks] Let $f(x) = x^2 + x + 8$ and $g(x) = 2x + 7$ be functions from the set of real numbers to the set of real numbers.   What is $g \circ f$?

Answer:

Question 3: [10 marks]

(a) Using the definition of **big-Omega notation**, show that $2x^3 + 4x^2 + 2$ is $\Omega(x^3)$.

> **Answer:**

(b) Analyze the time complexity of the following Python fragment, with x representing the problem size, and give a **Big-Oh estimate** of its running time. For the function g in your estimate $f(x)$ is $O(g)$, use a simple function g of smallest order. Justify your result.

```
sum = 0
for  i in range( 1, x // 2 ) :
        if  i % 3 == 0 :
                j = 2 * x
                while  j > 1 :
                        j = j / 5
                        sum += j
        else :
                for  k in range( 1,  4 * x ) :
                        sum += k
```

> **Answer:**

Question 4: [10 marks]

(a) $3 \equiv 9$ (mod 6) and $8 \equiv 14$ (mod 6). Therefore, which of the following are true?
Note that you can determine most of these without any calculations.

Answer:

    i) $3 + 36 \equiv 9 + 36$ (mod 6)

    ii) $3 + 14 \equiv 8 + 14$ (mod 6)

    iii) $8 / 2 \equiv 14 / 2$ (mod 6)

    iv) $(3)(8) \equiv (9)(14)$ (mod 6)

    v) $(3)(14) \equiv (9)(8)$ (mod 6)

(b) List five integers that are congruent to 3 modulo 19.

Answer: