Name:__Solutions__________________ Student Number:______________________________	CISC 203 Discrete Mathematics for Computing Science Test 1 Fall 2010 Professor Mary McCollam

This test is 50 minutes long and there are 40 marks.

Please write in pen and only in the box marked “Answer”.

This is a closed-book exam. No computers or calculators are allowed.

Write any assumptions you are making when answering a question.

Question 1:	/ 10
Question 2:	/ 10
Question 3:	/ 10
Question 4:	/ 10
Total:	/ 40

Question 1: [10 marks]

(a) Let $\mathcal{A} = \{x \mid -10 \le x \le 10\}$, $\mathcal{B} = \{x \mid -15 \le x \le 8\}$ and $C = \{x \mid 2 \le x \le 15\}$. Let the universe of discourse be $\mathcal{U} = \mathcal{Z}$, the set of integers. Determine the following set.

Answer:

$$(\mathcal{B} - C) \cap \mathcal{A} = \{ x \mid -10 \le x \le 1 \} = \{ -10, -9, -8, \ldots, 1 \}$$

Note: either of the above set descriptions is correct

(b) Let $\mathcal{A}_i = \{ \ldots, -2, -1, 0, 1, 2, \ldots, i \}$. Determine each of the following sets.

Answer:

i) $$\bigcup_{i=1}^{n} \mathcal{A}_i = \mathcal{A}_1 \cup \mathcal{A}_2 \cup \mathcal{A}_3 \cup \ldots \cup \mathcal{A}_n = \{ \ldots, -2, -1, 0, 1, 2, \ldots, n \}$$

ii) $$\bigcap_{i=1}^{n} \mathcal{A}_i = \mathcal{A}_1 \cap \mathcal{A}_2 \cap \mathcal{A}_3 \cap \ldots \cap \mathcal{A}_n = \{ \ldots, -2, -1, 0, 1 \}$$

Question 2: [10 marks]

(a) [4 marks] Determine whether the function $f(x) = 2x - 1$ is a bijection (one-to-one correspondence) from the set of positive integers to the set of positive integers Justify your answer.

Answer:

It is not a bijection because it is not onto. None of the even integers is an image under f.

There is no value x from the set of positive integers such that $f(x) = 2x - 1$ results in an even integer.

(b) [3 marks] What is the inverse of $f(x) = 6 - 3x^{1/2}$? You do not have to show that your result is correct.

Answer:

$f^{-1}(x) = ((6 - x) / 3)^2$

(c) [3 marks] Let $f(x) = x^2 + x + 8$ and $g(x) = 2x + 7$ be functions from the set of real numbers to the set of real numbers. What is $g \circ f$?

Answer:

$g \circ f(x) = g(f(x) = g(x^2 + x + 8) = 2(x^2 + x + 8) + 7 = 2x^2 + 2x + 23$

Question 3: [10 marks]

(a) Using the definition of ***big-Omega notation***, show that $2x^3 + 4x^2 + 2$ is $\Omega(x^3)$.

Answer:

For all $x > 1$, $4x^2 > 0x^3$ and $2 > 0x^3$

Therefore,

For all $x > 1$, $2x^3 + 4x^2 + 2 \geq 2x^3 + 0x^3 + 0x^3 = 2x^3$

Therefore, $2x^3 + 4x^2 + 2$ is $\Omega(x^3)$, since with witnesses $k = 1$ and $C = 2$,
for all $x > k$, $2x^3 + 4x^2 + 2 \geq Cx^3$

(b) Analyze the time complexity of the following Python fragment, with x representing the problem size, and give a ***Big-Oh estimate*** of its running time. For the function *g* in your estimate f(x) is O(*g*), use a simple function *g* of smallest order. Justify your result.

```
sum = 0
for  i in range( 1, x // 2 ) :
        if  i % 3 == 0 :
                j = 2 * x
                while  j > 1 :
                        j = j / 5
                        sum += j
        else :
                for  k in range( 1,  4 * x ) :
                        sum += k
```

Answer: Measure of input, is x; choose addition as key operation
Number of iterations of outer for loop: $\lfloor x / 2 \rfloor - 1$

Number of iterations of while loop inside if clause: ~ $\log_5 2x$
This while loop will be executed ~1/3 of the times through outer loop
Number of additions inside if clause is thus ~ $(1/3)(\log_5 2x)(\lfloor x / 2 \rfloor - 1)$ is O(xlogx)

Number of iterations of inner for loop inside else clause: 4x – 1
This while loop will be executed ~2/3 of the times through outer loop
Number of additions inside else clause is thus ~ $(2/3)(4x - 1)(\lfloor x / 2 \rfloor - 1)$ is $O(x^2)$

Worst case is $O(x^2)$. Thus T(x) is $O(x^2)$

Question 4: [10 marks]

(a) $3 \equiv 9 \pmod 6$ and $8 \equiv 14 \pmod 6$. Therefore, which of the following are true? Note that you can determine most of these without any calculations.

Answer:

i) $3 + 36 \equiv 9 + 36 \pmod 6$ TRUE

ii) $3 + 14 \equiv 8 + 14 \pmod 6$ FALSE

iii) $8 / 2 \equiv 14 / 2 \pmod 6$ FALSE

iv) $(3)(8) \equiv (9)(14) \pmod 6$ TRUE

v) $(3)(14) \equiv (9)(8) \pmod 6$ TRUE

(b) List five integers that are congruent to 3 modulo 19.

Answer:

Any of -16, -35, -54, -73, -92, … or 22, 41, 60, 79, 98, …