Name: ______________________________ Student Number: ____________________	CISC 203 Discrete Mathematics for Computing Science Test 3, Fall 2010 Professor Mary McCollam

This test is 50 minutes long and there are 40 marks. **Please write in pen and only in the box marked "Answer".** This is a closed-book exam. No computers or calculators are allowed.

NOTES:
Justify your answers to all of the counting problems (give explanation or show work).

All solutions with factorials only need to be reduced to factorial form, e.g., $\frac{12!\,5!}{2!\,4!}$

Question 1: [10 marks]

a) Show that if seven integers are selected from the first 10 positive integers, there must be at least two pairs of these integers with the sum 11.

Hint: Use the *Pigeonhole Principle*

Answer:

b) The name of a variable in the C programming language is a string that can contain uppercase letters, lowercase letters, digits, or underscores. Further, the first character in the string must be a letter, either uppercase or lowercase, or an underscore. If the name of a variable is determined by its first eight characters, how many different variables can be named in C? Note that the name of a variable may contain fewer than eight characters.

Answer:

Question 2: [10 marks] Use *mathematical induction* to prove that for every positive integer $n \geq 3$, $n^2 \geq 3n$.

Answer:

Question 3: [10 marks]

a) How many 18-digit bit strings contain exactly 6 0s and 12 1s if every 0 must be immediately followed by a 1? One such bit string is: 010101010101111111.

Answer:

b) How many different strings can be made from the letters in MOOSONEE, using all the letters?

Answer:

Question 4: [10 marks]

Let S be the subset of the set of ordered pairs of integers defined recursively by

Basis Step: $(0,0) \in S$

Recursive Step: If $(a,b) \in S$, then $(a+2, b+3) \in S$ and $(a+3, b+2) \in S$

a) [3 marks] List the elements of S produced by the first three applications of the recursive definition.

Answer:

b) [7 marks] Use *structural induction* to show that $5 \mid a+b$ when $(a,b) \in S$.

Answer: