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| Name: ______________________________ | CISC 203<br>Discrete Mathematics for Computing Science |
| Student Number: ______________________ | Test 4, Fall 2010 |
| | Professor Mary McCollam |

**Please write in pen and only in the box marked "Answer".**
This is a closed-book exam. No computers or calculators are allowed.

**Question 1: [10 marks**]

a) An employee joined a company in 2009 with a starting salary of $50,000. Every year this employee receives a raise of $1000 plus 5% of the salary of the previous year. Find a recurrence relation for the salary of this employee $n$ years after 2009.

**Answer:**

b) The following is the recurrence relation for a divide-and-conquer algorithm:

$$f(n) = 2\,f(n / 2) + 4$$

Use 'Theorem 1', given below, to construct a Big-Oh estimate for $f(n)$.
Show the intermediate steps.

**Theorem 1:** Let f be an increasing function that satisfies the recurrence relation $f(n) = a\,f(n / b) + c$, whenever $n$ is divisible by $b$, where $a \geq 1$, $b$ is an integer greater than 1, and $c$ is a positive real number. Then $f(n)$ is:

$O(n^{\log_b a})$ if $a > 1$
$O(\log n)$ if $a = 1$

**Answer:**

**Question 2: [10 marks]**

a) [8 marks] Determine whether the relation *R* on the set of all real numbers is reflexive, symmetric, asymmetric, and/or antisymmetric, where $(x,y) \in R$ if and only if $x = 2y$. Justify each answer with a brief explanation.

| **Answer:** |
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b) [2 marks] Let *R* be the relation on the set of people consisting of pairs (*a,b*), where *a* is a parent of *b*. Let *S* be the relation the set of people consisting of pairs (*a,b*) where *a* and *b* are siblings (brothers or sisters). What is $R \circ S$ ?

| **Answer:** |
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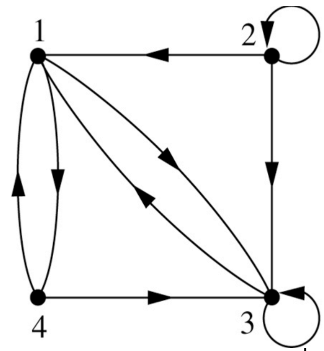


**Question 3: [10 marks]**

a) [4 marks] Determine whether the relation *R* represented by this directed graph is reflexive, symmetric, antisymmetric and/or transitive. Justify each answer with a brief explanation.

**Answer:**

b) [6 marks] Let $R_1$ and $R_2$ be relations on a set A = {1,2,3}.
$R_1$ = { (1,2), (2,1), (2,2), (2,3), (3,1) } and $R_2$ = { (1,2), (2,2), (2,3), (3,1), (3,2), (3,3) }.

**Answer:**

i) Represent each of these relations with a matrix with the elements of A listed in increasing order.

ii) Find the matrix that represents $R_1 \cap R_2$

iii) Find the matrix that represents $R_2 \circ R_1$

**Question 4: [10 marks]** Let ***R*** be the following relation on the set **{ x,y,z }**:

**{ (x,x), (x,z), (y,y), (z,x), (z,y) }**

Use the **0-1 matrix** representation for relations to find the transitive closure of ***R.*** Show the formula used to find the transitive closure of ***R*** from its 0-1 matrix representation and show the matrices in the intermediate steps in the algorithm, as well as the result matrix.

| **Answer:** |
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