Name:___SOLUTIONS_______________ Student Number:______________________________	CISC 203 Discrete Mathematics for Computing Science Test 1 Fall 2011 Professor Mary McCollam

This test is 50 minutes long and there are 40 marks. **Please write in pen and only in the box marked "Answer".**

This is a closed-book exam. No computers or calculators are allowed.

Question 1: [10 marks]

(a) [3 marks] Let A = { 0,2,4,6,8,10,12 }, B = { 1,2,3,4,5,6,7,8 }, and C = { 6,7,8,9,10,11 }. Determine each of the following sets:

Answer:

(i) $(B - A) \cap C = \{ 7 \}$

(ii) $(C \cap \overline{A}) \cup B = \{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 11 \}$

(b) [3 marks] Let A = { x | 0 < x < 4 } and B = { x | -3 < x < 0 } . Let the universe of discourse be $\mathcal{U} = \mathcal{Z}$, the set of integers. Find the Cartesian product A × B.

Answer:

{ (1, -2), (1, -1), (2, -2), (2, -1), (3, -2), (3, -1) }

(c) [4 marks] Determine whether each of the following sets is the power set of a set, where *a* and *b* are distinct elements. If the set is a power set, specify which set it is a power set of. If it is not a power set, explain why.

Answer:

(i) { ∅, {*a*} }

This is a power set of the set { *a* }

(ii) { ∅, {*a*}, {∅, *a*} }

This is not a power set, since it has 3 elements and the number of elements in a power set is 2^n, where n is the number of elements in the set.

Question 2: [10 marks]

(a) [4 marks] Determine whether $f : Z \times Z \rightarrow Z$ is a surjection (onto) if $f(m, n) = m + n + 1$. Justify your answer. Recall that Z is the set of integers.

Answer:

This is onto, since $f(0, n - 1) = n$ for every integer n

(b) [3 marks] Determine whether the function $f(x) = x^5 + 1$, where $f : Z \rightarrow Z$, is a bijection (one-to-one correspondence). Justify your answer.

Answer: No.

Justification:
It is not a surjection (onto), since many integers will not be in the range. For instance, f(1) = 2 and f(2) = 33 and the function is strictly increasing. So no integers between 2 and 33 will be in the range.

(c) [3 marks] What is the inverse of $f(x) = 3x^3 - 7$? You do not have to show that your result is correct.

Answer:

$f^{-1}(x) = ((x + 7) / 3)^{1/3}$

Question 3: [10 marks]

(a) Give a Big-Oh estimate for each of the following functions. For the function g of your estimate f(x) is O(g), use a simple function g of smallest order. Justify your result.

Answer:

(i) $(n^3 + n^2 \log n) (\log n + 1) + (17\log n + 19) (n^3 + 2)$

$O(n^3 \log n)$ Justification: The term n^3 dominates the first sum, logn the second, logn the third and n^3 the fourth. The product of the first two is therefore $O(n^3 \log n)$ and the product of the second two is the same. Therefore, the function is $O(n^3 \log n)$.

(ii) $(x^4 + x^2 + 1) / (x^4 + 1)$

O(1) Justification: The term x^4 dominates both sums, so the division results in 1.

(b) Analyze the time complexity of the following Python fragment, with x representing the problem size, and give a ***Big-Oh estimate*** of its running time. For the function *g* in your estimate *f*(x) is O(*g*), use a simple function *g* of smallest order. Justify your result.

```
y = 1
j = x * x
while  j > 1 :
        j = j / 2
        y = y * x
```

Answer:

$O(\log x^2)$

Measure of input, is x; choose multiplication as key operation
Number of iterations of while loop: $\sim \log_2 x^2$
Number of multiplications inside while loop is thus $\sim \log_2 x^2$

Question 4: [10 marks]

(a) Use the Euclidean algorithm to find gcd(2468, 8642).

Answer:

$8642 = 2468 \times 3 + 1238$

$2468 = 1238 \times 1 + 1230$

$1238 = 1230 \times 1 + 8$

$1230 = 8 \times 153 + 6$

$8 = 6 \times 1 + 2$

$6 = 2 \times 3 + 0$

gcd(2468, 8642) = 2

(b) $4 \equiv 10 \pmod 6$ and $14 \equiv 20 \pmod 6$. Therefore, which of the following are true? Note that you can determine most of these without any calculations.

Answer:

i) $4 + 10 \equiv 14 + 20 \pmod 6$ false

ii) $(4)(7) \equiv (14)(7) \pmod 6$ false

iii) $14 / 2 \equiv 20 / 2 \pmod 6$ false

iv) $(4)(20) \equiv (10)(14) \pmod 6$ true

v) $4 + 14 \equiv 10 + 20 \pmod 6$ true